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Analytical model for the design principle of large-area solar cells

Tetsuhiko Miyadera [a,*], Hiroyuki Ogo [a], Tetsuya Taima [a,b], Toshihiro Yamanari [a], Yuji Yoshida [a]

[a] Research Center for Photovoltaic Technologies, National Institute of Advanced Science and Technology (AIST), AIST Tsukuba Central 5, 1-1-1 Higashi, Tsukuba, Saitama 305-8565, Japan
[b] JST-PRESTO, Japan Science and Technology Agency (JST), 4-1-8 Honcho Kawaguchi, Saitama 332-0012, Japan

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ABSTRACT

A method for the analysis of large-area solar cells was established using an equivalent circuit model. A polygonal line approximation simplifies the model and the characteristics for large-area solar cells were formulated. The method determines the length limit of solar cells, which is dependent mainly on the sheet resistance of the transparent electrode and the open circuit voltage and slightly on the parasitic resistances such as series resistance and shunt resistance. Here we established a simple and quantitative design principle for large-area solar cells based on the model. This model can be applied to both organic and inorganic solar cells but is particularly useful for organic solar cells, whose characteristics are influenced by parasitic resistances.

1. Introduction

The development of organic solar cells (SCs) has come to the stage of fabricating large-area modules [1,2], and analysis of large-area organic SCs has been reported [3–5]. The fill factor (FF) and short circuit current density (J_{sc}) are degraded with increase in the area of SCs, due to the increase in resistance of the transparent electrode. Various techniques, such as integration of bus bar electrodes and modularization by scribing, have been adopted to prevent degradation by reduction of the effective resistance of the transparent electrodes.

Design principles for the fabrication of large-area SCs are required that would describe the relationship between the length scale of the device and its characteristics. There have been some reports regarding the size-dependent characteristics of SCs based on model analyses that have dealt with power loss at the transparent electrode [3], whereas others have dealt with discrete series resistance [4]. However, some important factors were ignored, such as the shunt resistance or diode component. Such parameters should be integrated for the quantitative analysis of length-scale SCs.

In the field of inorganic SCs, large-scale SCs are often analyzed using a distributed-constant equivalent circuit model [6–10], where elements such as diodes and resistors are assumed to be distributed continuously (Fig. 1(a)). The model enables the quantitative analysis of the length-scale characteristics of the SC; however, the model cannot be analytically solvable, because of a nonlinear component that comes from the diodes and the feature of implicit function that comes from series resistance. Numerical calculations or approximation methods are therefore used for analysis of the model. The model is desired to be applied to organic SCs, whose characteristics are strongly influenced by parasitic resistances, such as series and shunt resistances. Therefore, an approximation method dealing with the parasitic resistances is required for application of the model to organic SCs. In this study, a novel approximation method is proposed for the quantitative analysis of length-scale SC characteristics. The model can be applied to both organic and inorganic SCs.

2. Models and formulation

The equivalent circuit consists of a current source (i_L), diode (i_d), shunt resistance (r_{sh}), series resistance (r_c), and the sheet resistance of the transparent electrode (ρ_s) (Fig. 1(a)). The length of the device is set as L and the width of the device as unity because the circuit equation and results are independent of the width. When the current flow and voltage distribution are considered (Fig. 1(b)), the circuit equation can be expressed as

$$\frac{d^2V(x)}{dx^2} = \frac{\rho_s}{r_{sh}+r_c}\{V(x)+r_{sh}i_d-r_{sh}i_L\}. \quad (1)$$

The current density–voltage (J–V) characteristics of the diode are usually expressed using an exponential function

$$i_d = i_0\{\exp(qV_d/nk_BT)-1\}, \quad (2)$$

where $V_d = V(x)-r_c(di/dx)$. This nonlinear implicit equation can be simplified by approximating the diode characteristics with a polygonal line, as shown in Fig. 1(c):

$$i_d = g_{sc}V_d \quad (V \le V_p),$$

* Corresponding author. Tel.: +81 29 861 0536; fax: +81 29 861 6232.
E-mail address: tetsuhiko-miyadera@aist.go.jp (T. Miyadera).

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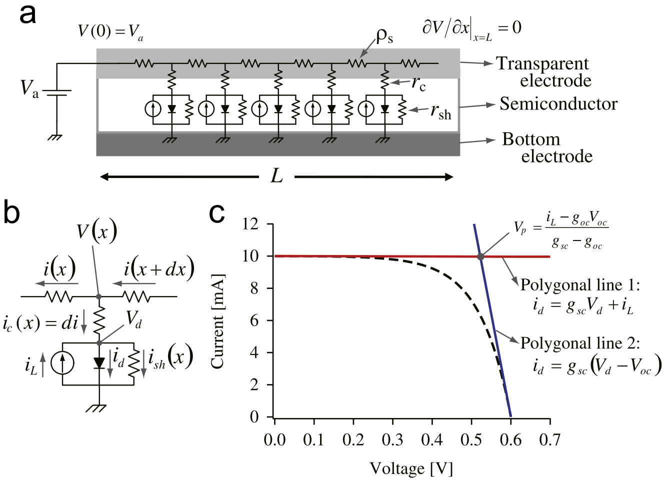

Fig. 1. (a) Equivalent circuit model for large-scale SCs using a distributed constant circuit. (b) Diagram of the current flow and voltage around the point x of the equivalent circuit. (c) Typical SC characteristics (dotted line) and polygonal line approximation (solid line).

$$i_{\mathrm{d}} = g_{\mathrm{oc}}(V_{\mathrm{d}} - V_{\mathrm{oc}}) + i_{\mathrm{L}} \quad (V_{\mathrm{p}} \le V), \tag{3}$$

where $g_{\mathrm{sc}} = \partial i_{\mathrm{d}}/\partial V|_{V=0}$, $g_{\mathrm{oc}} = \partial i_{\mathrm{d}}/\partial V|_{V=V_{\mathrm{oc}}}$, and $V_{\mathrm{p}} = (i_{\mathrm{L}} - g_{\mathrm{oc}}V_{\mathrm{oc}})/(g_{\mathrm{sc}} - g_{\mathrm{oc}})$. With this polygonal line approximation (PLA), Eq. (1) can be transformed to

$$\frac{d^2V(x)}{dx^2} - \frac{\rho_s}{r_{\mathrm{sh}} + r_{\mathrm{c}} + r_{\mathrm{sh}}r_{\mathrm{c}}g_{\mathrm{sc}}}\{(1 + r_{\mathrm{sh}}g_{\mathrm{sc}})V(x) - r_{\mathrm{sh}}i_L\} = 0 \quad (V \le V_{\mathrm{p}}),$$

$$\frac{d^2V(x)}{dx^2} - \frac{\rho_s}{r_{\mathrm{sh}} + r_{\mathrm{c}} + r_{\mathrm{sh}}r_{\mathrm{c}}g_{\mathrm{oc}}}\{(1 + r_{\mathrm{sh}}g_{\mathrm{oc}})V(x) - r_{\mathrm{sh}}g_{\mathrm{oc}}V_{\mathrm{oc}}\} = 0 \quad (V_{\mathrm{p}} \le V). \tag{4}$$

This approximation makes the equation analytically solvable, even with the existence of series resistance. The equations are solved under boundary conditions of $V(0) = V_{\mathrm{a}}$ and $dV/dx|_{x=L} = 0$, where V_{a} is the applied voltage. Here, $x = 0$ is defined as the point where the collection electrode is connected, whereas $x = L$ is defined as the opposite edge of the device. The form of the solution is divided by the conditions of L and V_{a}.

(i) When L and V_{a} are small, $V(x) \le V_{\mathrm{p}}$ is valid for all x. The solution is

$$V(x) = A + (V_{\mathrm{a}} - A)(\cosh(s_1(L-x)))/\cosh(s_1L), \tag{5}$$

$$i = \partial V/\partial x|_{x=0}/\rho_s = (s_1(V_{\mathrm{a}} - A)\tanh(s_1L))/\rho_s, \tag{6}$$

where $A = i_L/((1/r_{\mathrm{sh}}) + g_{\mathrm{sc}})$ and $s_1 = \sqrt{\rho_s(1 + r_{\mathrm{sh}}g_{\mathrm{sc}})/(r_{\mathrm{sh}} + r_{\mathrm{c}} + r_{\mathrm{sh}}r_{\mathrm{c}}g_{\mathrm{sc}})}$.

(ii) When L is large, $V(x) \le V_{\mathrm{p}}$ is valid for x larger than a certain value, x_{p}. The solution is

$$V(x) = A + \frac{(V_{\mathrm{a}} - A)\{s_1P_3\cosh(s_1(x - x_{\mathrm{p}})) + s_3P_4\sinh(s_1(x - x_{\mathrm{p}}))\} - (B - A)s_3P_4\sinh(s_1x)}{s_1P_1P_3 - s_3P_2P_4} \quad (0 \le x \le x_{\mathrm{p}}),$$
$$V(x) = B + \frac{s_1\{(V_{\mathrm{a}} - A) - (B - A)P_1\}\cosh(s_3(x - L))}{s_1P_1P_3 - s_3P_2P_4} \quad (x_{\mathrm{p}} \le x), \tag{7}$$

$$i = \frac{s_1}{\rho_s}\frac{(V_{\mathrm{a}} - A)(s_3P_1P_4 - s_1P_2P_3) - (B - A)s_3P_4}{s_1P_1P_3 - s_3P_2P_4}, \tag{8}$$

where $B = (g_{\mathrm{oc}}V_{\mathrm{oc}})/((1/r_z) + g_{\mathrm{oc}})$, $s_3 = \sqrt{\rho_s(1 + r_{sh}g_{oc}V_{oc})/(r_{sh} + r_c + r_{sh}r_cg_{oc})}$, $P_1 = \cosh(s_1x_{\mathrm{p}})$, $P_2 = \sinh(s_1x_{\mathrm{p}})$, $P_3 = \cosh(s_3(x_{\mathrm{p}} - L))$, and $P_4 = \sinh(s_3(x_{\mathrm{p}} - L))$. The length L_{p} that divides conditions (i) and (ii) is expressed as

$$L_{\mathrm{p}} = \frac{1}{s_1}\ln\left[(V_{\mathrm{a}} - A)/(V_{\mathrm{p}} - A) + \sqrt{\{(V_{\mathrm{a}} - A)/(V_{\mathrm{p}} - A)\}^2 - 1}\right]. \tag{9}$$

(iii) When V_{a} is larger than V_{p}, $V(x) \ge V_{\mathrm{p}}$ is valid for all x. The solution is

$$V(x) = B + (V_{\mathrm{a}} - B)(\cosh(s_3(L-x)))/\cosh(s_3L), \tag{10}$$

$$i = (s_3(V_{\mathrm{a}} - B)\tanh(s_3L))/\rho_s. \tag{11}$$

The length-scale J–V characteristics of SCs can be calculated by substituting the parameters in the appropriate formula. However, x_{p} cannot be calculated analytically, but it can be approximately calculated using the second-order Newton method:

$$x_{\mathrm{p}} \approx L_{\mathrm{p}} + \left.\frac{dx_{\mathrm{p}}}{dV}\right|_{x_{\mathrm{p}} = L_{\mathrm{p}}}(V_{\mathrm{p}} - V(L_{\mathrm{p}})) + \frac{1}{2}\left.\frac{d^2x_{\mathrm{p}}}{dV^2}\right|_{x_{\mathrm{p}} = L_{\mathrm{p}}}(V_{\mathrm{p}} - V(L_{\mathrm{p}}))^2$$
$$= L_{\mathrm{p}} + \left(\left.\frac{dV}{dx_{\mathrm{p}}}\right|_{x_{\mathrm{p}} = L_{\mathrm{p}}}\right)^{-1}(V_{\mathrm{p}} - V(L_{\mathrm{p}})) - \frac{1}{2}\left\{\left.\frac{d^2V}{dx_{\mathrm{p}}^2}\right|_{x_{\mathrm{p}} = L_{\mathrm{p}}}\left(\left.\frac{dV}{dx_{\mathrm{p}}}\right|_{x_{\mathrm{p}} = L_{\mathrm{p}}}\right)^{-3}\right\}$$
$$\times (V_{\mathrm{p}} - V(L_{\mathrm{p}}))^2. \tag{12}$$

3. Calculation results

The L-dependent J–V characteristics were calculated (Fig. 2(a)) based on a PLA and using parameters typical for organic SCs (Table 1). Numerical calculations without the PLA were also conducted (Eq. (1) with Eq. (2)) for comparison. The trend of J_{sc} and FF, which were calculated using both methods (Fig. 2(b)), is the same as that for L, so that the validity of the PLA is verified.

When L is small, the fill factor decreases monotonically and J_{sc} remains constant. With increase of L higher than a certain value (L_{p}), the decrease of FF becomes saturated and J_{sc} starts to decrease. In contrast, V_{oc} is independent of L. The decrease of FF can be explained by the increase of series resistance with the increase of L. The trend of J_{sc} can be explained as follows. When L is small and $V(x) < V_{\mathrm{p}}$ is valid at any position, the current flows mainly into the transparent electrode rather than the diode. Under this condition, carriers can be collected effectively and J_{sc} is kept constant. When L becomes larger than L_{p}, $V(x) > V_{\mathrm{p}}$ is valid at points farther than x_{p} ($\sim L_{\mathrm{p}}$). The current flows into the diode rather than the transparent electrode, which causes a decay of J_{sc}. Therefore, to prevent the decrease in J_{sc}, the length of the unit cell of a module should be less than L_{p}. L_{p} can then be used as an index for the module design.

3.1. ρ_s dependence

The trend of J_{sc} and FF as functions of L (Fig. 3(a, b)) is strongly dependent on ρ_s. According to Eq. (9), L_{p} is inversely proportional to the square root of ρ_s, which was confirmed by the results shown in Fig. 3(c). If the typical value of ρ_s for indium tin oxide (ca. 10 Ω/sq) and typical organic-SC parameters are substituted, then an L_{p} value of