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Novel variational formulations for nonlocal plasticity

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Abstract

A nonlocal structural model of softening plasticity is considered in the framework of the internal variable theories of inelastic behaviours of associative type. The finite-step nonlocal structural problem in a geometrically linear range is formulated according to a backward difference scheme for time integration of the flow rule. The related finite-step variational formulation in the complete set of local and nonlocal state variables is recovered. A family of mixed nonlocal variational formulations, with different combinations of state variables, is provided starting from the general variational formulation. The specialization of a mixed variational formulation to existing nonlocal models of softening plasticity, assuming both linear and nonlinear constitutive behaviour, is provided to show the effectiveness of the theory.

Keywords: Nonlocal plasticity; Structural models; Variational formulations

1. Introduction

It is well known that conventional plasticity theories are unable to describe softening since they give rise to ill-posed boundary value problems after the onset of localization.

A material length scale emerges in many problems such as in the development of shear bands and fracture. In a shear band the deformation is localized within a confined region as a result of strain softening and the thickness of the shear band is finite and is set by the

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microstructure of the material. Standard plasticity models cannot describe the localization of plastic deformation in a finite band due to the lack of a length scale which provides the width of the shear band (see e.g. de Borst et al., 1993; Engelen et al., 2006).

Nonlocal theories have been initially developed in the context of elasticity in order to regularize singular stress fields (Kroner, 1967; Eringen et al., 1977). Then nonlocal plasticity theories have been developed for the purpose of regularising localization of deformation as a result of material softening. The link between nonlocality and regularisation was made in Bažant et al. (1984), Pijaudier-Cabot and Bažant (1987) in a continuum damage context in which the variable which controls the damage growth is replaced by its weighted average in order to spread damage growth thus avoiding pathological localization.

Continuum-based nonlocal theories and treatments based on the modelling of discontinuities are two different approaches in the existing literature on localized failure mechanisms. Representatives of the first category are viscoplastic regularization techniques, nonlocal and gradient theories and micropolar Cosserat models. In order to overcome the deficiencies of classical models, nonlocal plasticity theories have been proposed among others by Bažant and Lin (1988), Leblond et al. (1994), Vermeer and Brinkgreve (1994), Strömber and Ristinmaa (1996), Needleman and Tvergaard (1998), Svedberg and Runesson (1998), Borino et al. (1999), Borino and Failla (2000), and Jirásek and Rolshoven (2003). Higher-order plasticity models can be found in, e.g. Dillon and Kratochvil (1970), Aifantis (1987), de Borst and Pamin (1996), de Borst (2001), Fleck and Hutchinson (2001), Saczuk et al. (2003) and Baek and Srinivasa (2003) in which the free energy depends not only on plastic strain but also on spatial gradient, the governing equations are obtained by means of a variational procedure based on the maximum dissipation rate and the issue of additional boundary conditions is discussed.

The second group is given by finite element approaches and mesh processors with embedded localized zones with discontinuities in which a characteristic length scale appears on the numerical level by relating the discontinuity to a given mesh size (see e.g. Miehe and Schröder, 1994; Garikipati and Hughes, 2000; Carstensen et al., 2005; Carstensen, 2007).

In nonlocal theories endowed with internal variables, spatial interactions at a certain length scale are introduced with reference to internal variables which control the evolution of inelastic phenomena. These interactions are strongly nonlocal since they act over finite distances and this fact is particularly important when modelling fracture where strong nonlocality is needed to deal with the singularities at a crack tip (Peerlings et al., 2002).

The broaden problem of providing variational formulations for the nonlocal elastoplastic structural problem has received large attention. Contributions in this field can be found in e.g. Borino et al. (1999), Marotti de Sciarra (2004) for rate nonlocal plasticity and in Mühlhaus and Aifantis (1991) for gradient plasticity. A different approach to localization problems is based on a constitutive minimization principle for standard dissipative materials that optimizes a generalized incremental work function with respect to internal variables. Localization phenomena are interpreted as microstructure developments on multiple scales in analogy to elastic phase decomposition problems (Miehe and Lambrecht, 2003; Miehe et al., 2004). A method involving the relaxation of variational incremental problems which are derived from an elastic and a plastic potential is proposed in Mielke (2004) and Bartels et al. (2006). From a mathematical perspective, variational models for continua allowing for softening and fracture are addressed in Francfort and Marigo (1998), Braides et al. (1999), and Dal Maso et al. (2005) and for damage in