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# Size-dependent bending of thin metallic films

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**Abstract**

Size-dependent large curvature pure bending of thin metallic films has been analytically studied taking into account the associated strengthening mechanisms at different thickness scales. The classical plasticity theory is applicable to films thicker than 100 μm. Consequently, their bending capacity is governed by the competition between the material hardening and the thickness reduction. For films with a thickness ranging from fractions of a micron to a few microns, in addition to the above mechanisms, the strain gradient effect plays an important role and introduces an internal length scale. When the film thickness reduces to the nano-scale, the strain gradient effect is gradually replaced by the dominant surface stress/energy effect.


*Keywords:* Strain gradient plasticity; Intrinsic length scale; Surface effect; Nano-plate

## 1. Introduction

The theory of the plastic deformation of plates (whose thickness is larger than 0.1 mm) has been well established (Hill, 1950). The bending moment of a metallic plate, normalized by the maximum elastic bending moment, increases with the bending curvature, normalized by the initial plate thickness, reaches a maximum at a critical non-dimensional bending curvature, and drops with the further increment of the non-dimensional bending curvature (Rivlin, 1949; Chen, 1962; Triantafyllidis, 1980; Triantafyllidis and Needleman, 1982; Zhu, 2007). The value of the critical non-dimensional bending curvature depends on

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the competition between the material strain hardening and the plate thickness reduction. Zhu, 2007 has found that the smaller the material strain hardening exponent, the smaller the critical non-dimensional bending curvature. For a conventional plate made of an elastic perfectly plastic (i.e. the strain hardening exponent is 0) metallic material, the critical non-dimensional bending curvature is 0.01–0.15, depending on the material's yield strain. If the material strain hardening exponent is 0.3, the critical non-dimensional bending curvature is between 0.5 and 1.5. Zhu, 2007 has demonstrated that when a shell structure is crushed, the maximum non-dimensional bending curvature can well exceed 1.5.

Applications of metallic and polymeric materials at micron scale are multiplying rapidly. The existence of an internal length scale has been found and studied for several decades (Dillon and Kratochvil, 1970; Aifantis, 1987; Fleck and Hutchinson, 1993; Fleck et al., 1994; Nix and Gao, 1998; Stolken and Evans, 1998; Gao et al., 1999; Huang et al., 2000; Hutchinson, 2000; Haque and Saif, 2003; Lam et al., 2003; Wang et al., 2003; Abu Al-Rub and Voyiadjis, 2006; Wang et al., 2007). It is generally believed that when the thickness is at micron scale, strain gradient can greatly affect the bending stiffness of thin films. It is also believed that strain gradient effect does not exist in nano-scale metallic structures. The intrinsic material length of strain gradient effect is usually about a few microns and it varies from material to material. To the best of our knowledge, when predicting a relationship between the bending moment and the bending curvature for micro-plates or micro-beans, all the available models and analyses ignore the thickness reduction of the bent plates or beams (Huang et al., 2000; Wang et al., 2003; Abu Al-Rub and Voyiadjis, 2006). However, large curvature deformations can result in significant thickness reduction (Hill, 1950; Chen, 1962; Triantaflyllidis, 1980) and the bending stiffness depends upon the competition between the material strain hardening and the plate thickness reduction (Zhu, 2007). In this paper, we will examine the combined effect of material strain hardening, strain gradient increment and the thickness reduction on the bending stiffness of micro-plates (or films).

The mechanical properties of nanomaterials and nano-structures have been attracting more and more attention in recent years. As the surface to volume ratio is large, nano-structures can present exceptional properties (He et al., 2004; Lim and He, 2004; Cuenot et al., 2004; Zhou and Huang, 2004; Duan et al., 2005a,Duan et al., 2005b,Duan et al., 2005c; Wang et al., 2006). Although it is difficult to fabricate nano-plates and nano-beams, it is eventually unavoidable that nano-plates and nano-beams will be present in various nano-technologies (Zhou and Huang, 2004). It has been well documented that the surface area of a bent plate increases with the bending curvature (Rivlin, 1949; Hill, 1950; Chen, 1962; Triantaflyllidis, 1980; Triantaflyllidis and Needleman, 1982; Zhu, 2007). One of the aims of this paper is to examine the contribution of the surface energy to the effective bending capacity of a bent metallic nano-plate.

## 2. Pure bending of macro-plates

The analytical work on the bending of plates has been well documented (Rivlin, 1949; Chen, 1962; Triantaflyllidis, 1980; Triantaflyllidis and Needleman, 1982; Zhu, 2007). In this section, we briefly introduce the analysis and the main results of the pure bending of macro-plates, as we need to make use of the related formalism and to compare the results of the macro-plates with those of micro- and nano-plates in the subsequent sections.