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Basic features of plastic strains: From micro-mechanics to incrementally nonlinear models

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Abstract

Most rate-independent constitutive relations for granular materials are based on the existence of a regular flow rule. This assumption states that once the mechanical state of a material point belongs to the yield surface, then the direction of the plastic strains is independent of the loading direction. In this paper, the notion of a regular flow rule is shown to exist only for two-dimensional and axisymmetric loading conditions. By considering our incrementally nonlinear constitutive model, it is established that this notion disappears as soon as more general loading conditions are applied, as also predicted from discrete element simulations. Moreover, a sound micro-mechanical interpretation of the vanishing of a regular flow rule in three-dimensional loading conditions is given from a multi-scale perspective using the micro-directional model. This model highlights the great influence of the loading history on the shape of the plastic Gudehus response-envelope.

Keywords: Elastoplasticity; Granular material; Discrete element method; Micro-mechanics; Multi-scale modeling; Incremental nonlinearity; Flow rule

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1. Introduction

A great deal of materials can be considered to a large extent as granular materials because a scale exists at which they exhibit a discrete structure made up of an assembly of well-identified particles that interact with one another by cohesive–frictional forces. For example, geomaterials such as rock assembly or sands belong to this class of materials. Modeling the mechanical behavior of granular materials is of paramount importance to conducting large-scale engineering computations. For instance, the design of civil engineering structures often requires robust, realistic constitutive equations for soils that are able to describe the response of an elementary specimen of soil along various loading paths. At the same time, the development of powerful numerical techniques such as the finite element method, together with the rapid improvement of computer capacity, have warranted building adequate constitutive models. One way currently adopted to develop constitutive relations is to formulate, through a suitable mathematical formalism, the physical features revealed from observations (laboratory experimental tests, etc.). This approach leads to the broad class of phenomenological models. Subsequent constitutive relations link both kinematic and static variables at the mesoscopic scale, which corresponds to the elementary representative volume, in which average variables (namely, stress and strain variables) can be defined. This is also the scale of the finite element cell. In this type of approach, the discrete nature of the materials is not explicitly accounted for. Their discrete nature can indirectly appear through internal parameters such as grain size or void ratio. One subsequent drawback inherent to this class of models is that the outstanding features of the constitutive behavior of materials (normality rule, hardening mechanism, etc.) are assumed a priori. Thus, these models cannot investigate whether a given property remains valid irrespective of the loading conditions, and above all, it cannot determine what the micro-mechanical origin of this property is.

This paper focuses on a question currently raised on the mechanical behavior of materials, namely the existence of a regular flow rule. After having reviewed in the second section how a flow rule is defined in the framework of standard elastoplasticity, this notion is examined in the third section by considering the framework of incrementally nonlinear constitutive models. Then the last section is devoted to micro-mechanical approaches with both discrete element modeling (the molecular dynamics method) and the micro-directional model. This latter model, based on a homogenization scheme, gives rise to a micro-structural interpretation of the notion of a flow rule by relating a macroscopic phenomenon (the existence of a regular or a singular flow rule) to microscopic mechanisms at the grain (or grain contact) scale. It is worth emphasizing that the conclusions drawn in this paper can be extended to a wide range of materials, to the extent that a scale on which a discrete microstructure exists can generally be exhibited. Geomaterials and related constitutive models are considered for the sake of illustration.

Throughout this paper, only rate-independent materials are considered under static loading conditions, so that dynamical effects can be omitted. The existence of a regular (or singular) flow rule is investigated in this context. Thus, as discussed by Sandler and Rubin (Sandler and Rubin, 1987), this context precludes the occurrence of spontaneous disturbances (leading, for example, to a nonunique solution), even though nonassociated flow rules are considered. With no additional prescription, the summation convention on repeated indices will be employed. Moreover, for any (one- or two-order) tensor A, ${}^{t}A$ denotes the transpose tensor.