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# An elasto-plastic constitutive model for evolving asymmetric/anisotropic hardening behavior of AZ31B and ZEK100 magnesium alloy sheets considering monotonic and reverse loading paths

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ABSTRACT

Deformation twinning and texture evolution in hexagonal close-packed (HCP) metals lead to evolving flow stress asymmetry/anisotropy and evolving plastic anisotropy. These phenomena cause a significant change in the shape of the yield surface with accumulated plastic deformation which cannot be modeled accurately with traditional hardening laws. In this paper, an anisotropic continuum-based plasticity model is proposed to capture the large strain cyclic hardening behavior of magnesium alloys. Key in the current formulation is the incorporation of distortional hardening to model the evolving asymmetric/anisotropic hardening response of magnesium alloys for both monotonic and reverse loading paths. The hardening behavior is classified into three deformation modes: Monotonic Loading [ML], Reverse Compression [RC], and Reverse Tension [RT]. The deformation modes correspond to the different loading regimes of the cyclic hardening curve. Specifically, the ML mode corresponds to the initial in-plane tension and the initial in-plane compression from the annealed state, the RC mode corresponds to the in-plane compression following previous tension and the RT mode corresponds to the in-plane tension following previous compression. Three separate hardening laws are used to define the hardening response within each deformation mode. Moreover, a multi-yield surface modeling approach is used where a CPB06 type anisotropic yield surface is assigned to each deformation mode. The evolution of the anisotropy coefficients involved in the expression of the yield function, is considered to model distortional hardening within each deformation mode. The evolving anisotropy parameters are found by minimizing the difference between the model predictions and the experiments, together with the interpolation technique proposed by Plunkett et al. (2006). The proposed model is calibrated using monotonic and reverse loading experimental data for AZ31B and ZEK100 magnesium alloys. A strain rate independent elasto-plastic formulation is used to implement the proposed constitutive model as a user material subroutine (UMAT) in the commercial finite element software LS-DYNA®. The predictions of the model are compared against the experimental monotonic and cyclic

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(CTC and TCT) flow stresses and r-values of AZ31B and ZEK100 sheets along different test directions. An excellent agreement is found between the simulated and experimental results.



## 1. Introduction

Recent efforts by the automotive industry to lower the fuel consumption and reducing $CO_2$ emissions, have led to the use of lightweight materials in vehicles. Magnesium (Mg) alloys being the lightest of all possible structural engineering metals, have attracted a lot of scientific and technical interests due to their low density, superior specific tensile strength and rigidity compared to steel and aluminum alloys (Roberts, 1960; Avedesian and Baker, 1999). Currently, the majority of magnesium parts used in automotive applications are fabricated by die casting. The die cast magnesium parts often have poor fatigue strength and ductility (Doege and Dröder, 2001). Wrought magnesium alloys, on the other hand, typically exhibit superior mechanical properties such as tensile and fatigue resistance (Bettles and Gibson, 2005; Agnew et al., 2006). However, manufacturing with wrought magnesium alloys has been limited due to their poor formability at room temperature (Avedesian and Baker, 1999).

The mechanical properties of magnesium alloys can vary significantly depending on the initial texture and loading path. Metal forming processes such as rolling can induce strong textures in wrought magnesium alloys. A well-known example of this is the AZ31B magnesium sheet, which usually has a strong basal texture developed due to prior rolling (Kaiser et al., 2003; Barnett et al., 2004a,b; Styczynski et al., 2004), where the majority of grains have their *c*-axes aligned parallel to the sheet normal direction (ND) (Yukutake et al., 2003). Thus, any stress-state, which imposes compressive strains in the sheet normal direction would require contraction along the *c*-axes of majority of the grains. However, contraction along the *c*-axes cannot be accommodated by slip of *a* type dislocations and requires the activation of pyramidal $c + a$ slip systems or $\{10\bar{1}1\}\{10\bar{1}\bar{2}\}$ contraction twins (Obara et al., 1973; Yoo, 1981; Knezevic et al., 2010). These systems are much harder to activate at room temperature due to their high CRSS (Yoo, 1981; Gall et al., 2013). On the contrary, in-plane compressive loading results in extension of the *c*-axis of the lattice. This tensile strain along the *c*-axis can be accommodated by easily activated $\{10\bar{1}2\}\{\bar{1}011\}$ extension twinning at room temperature (Agnew and Duygulu, 2005). This strong dependence of deformation mechanisms on the in-plane loading direction of AZ31B sheet leads to a strong tension-compression yield asymmetry at room temperature (Gall et al., 2013). In addition, the limited formability of AZ31B sheet has also been linked to the strong basal texture, which offers only a limited number of favorably oriented slip systems for plastic deformation at room temperature (Agnew and Duygulu, 2005; Lou et al., 2007). It has been shown that the room temperature formability of rolled magnesium alloys can be improved by weakening the basal texture with addition of Rare-Earth (RE) elements (Bohlen et al., 2007; Chino et al., 2010; Dreyer et al., 2010; Al-Samman and Li, 2011). Huang et al. (2012) has studied the effects of the addition of RE elements on textural evolution and mechanical properties during post annealing of an Mg-4Y-3RE magnesium alloy sheet processed by differential speed rolling at 823 K. In their work, it was suggested that the addition of RE elements promotes static recrystallization at pre-existing grain boundaries due to solute effects of rare-earth elements, resulting in randomization of basal texture. The weakened basal texture enhances the activity of non-basal slips resulting in improved formability of these alloys (Chino et al., 2010).

Deformation twinning can significantly influence the in-plane hardening response of wrought Mg alloys (Lou et al., 2007; Knezevic et al., 2010). The compressive hardening behavior of Mg sheet is characterized by an S-shaped sigmoidal hardening curve exhibiting a low initial yield stress (Nobre et al., 2002) followed by a concave-up stress-strain behavior with a low initial hardening rate due to extension twinning (Yukutake et al., 2003). At large compressive strain, due to the exhaustion of twinning and dominance of slip mechanisms, the stress strain curve switches to that of a typical concave-down shape (Yukutake et al., 2003; Lou et al., 2007). Apart from slip and twinning, untwinning may occur in a previously twinned material and can be characterized by the disappearance of existing twin bands (Lou et al., 2007). Untwinning can occur during reverse loading paths such as in-plane tension following previous in-plane compression and results in an inflected S-shaped flow curve similar to that of twinning (Lou et al., 2007; Wu et al., 2008). As mentioned previously, extension twinning is expected during in-plane compression of wrought magnesium alloys (such as AZ31B), since the loading axis is perpendicular to the c-axes of majority of grains. Extension twinning results in an 86.3° reorientation of basal poles towards the in-plane loading direction (Wang et al., 2012). Hence, during in-plane reverse tension, the loading axis is parallel to the c-axes of the previously twinned crystals during in-plane compression. This causes the previously twinned crystals to undergo a second extension twinning event usually referred to as untwinning/detwinning. However, this second twinning (extension) event can proceed on any of the six available twinning planes depending on the critical resolved shear stress where only one of them would reorient the twinned zone back to the parent orientation. Other variants of the twin would give rise to new twinned orientations, which may be referred to as retwinning. However, for simplicity, the term untwinning has been used to refer to both untwinning and retwinning in the present work. In cyclic loading of Mg alloys at room temperature, twinning and untwinning appear alternately and lead to a large asymmetry of cyclic deformation (Lou et al., 2007; Lee et al., 2008). The asymmetric loading reversals and the Bauschinger effect are often found to become more significant with increasing strain amplitudes (Xiong et al., 2013).