International Journal of Plasticity 70 (2015) 98–125

Contents lists available at ScienceDirect

International Journal of Plasticity

journal homepage: www.elsevier.com/locate/ijplas

Modeling and algorithms for two-scale time homogenization of viscoelastic-viscoplastic solids under large numbers of cycles

S. Haouala[*], I. Doghri

Université Catholique de Louvain (UCL), Institute of Mechanics, Materials and Civil Engineering (IMMC), Bâtiment Euler, 4 Avenue G. Lemaître, B-1348 Louvain-la-Neuve, Belgium

ARTICLE INFO

Article history:
Received 19 December 2014
Received in revised form 10 March 2015
Available online 25 March 2015

2010 MSC:
00-01
99-00

Keywords:
Time homogenization
B. Cyclic loading
B. Viscoelastic (VE) – viscoplastic (VP) material
C. Numerical algorithms
C. Asymptotic analysis

ABSTRACT

A two-scale time homogenization formulation and the corresponding algorithms are proposed for coupled viscoelastic–viscoplastic (VE–VP) homogeneous solids subjected to large numbers of cycles. The main aim is to predict the long time response while reducing the computational cost considerably. The method is based on the definition of macro and micro-chronological time scales, and on asymptotic expansions of the unknown variables. First, the VE–VP constitutive model is formulated based on a thermodynamical framework. Next, the original VE–VP initial-boundary value problem is decomposed into coupled micro-chronological (fast time scale) and macro-chronological (slow time-scale) problems. The former is purely VE, and solved once for each macro time step, whereas the latter problem is nonlinear and solved iteratively using fully implicit time integration. For micro-scale time averaging, one-point and multi-point integration algorithms are developed. Several numerical simulations on uniaxial and multiaxial cyclic loadings illustrate the computational efficiency and the accuracy of the proposed methods.

1. Introduction

Nowadays, polymer materials contribute to the improvement of properties and reliability of many components. They find widespread use in various industries such as aerospace, automotive and consumer electronics. During service, many of these polymer-based parts are exposed to cyclic loading conditions resulting in the degradation of material properties. The design of these complex components or structures still requires expensive testing and experiments which are generally limited to small structural components. Therefore the prediction of the long-term behavior of these structures involves some sort of modeling, and it requires significant computational resources, in particular, for cyclic loadings when taking into account the nonlinear behavior of the polymer materials.

In fact, polymeric materials such as thermoplastics exhibit a complex inelastic behavior and they are time and rate-dependent at all stages. Zhang and Moore (1997) show the behavior of High Density Polyethylene (HDPE) under uniaxial compression tests. The behavior of the material is inelastic for both loading and unloading and it depends on both strain and strain rate. Thereby proper engineering design with these materials is subject to the development of an adequate modeling

---

[*] Corresponding author. Tel.: +32 10 472354/472350; fax: +32 10 472180.
E-mail addresses: sarra.haouala@uclouvain.be (S. Haouala), issam.doghri@uclouvain.be (I. Doghri).

http://dx.doi.org/10.1016/j.ijplas.2015.03.005
0749-6419/

able to reproduce their mechanical behavior and also the possibility to simulate it numerically. In the literature, some authors describe the behavior of polymer materials by nonlinear viscoelastic (VE) models for instance (Khan et al. (2006), Ayoub et al. (2011) and Zari et al. (2011)) by using a combination of linear and nonlinear springs with dashpots. Several elasto-viscoplastic (EVP) constitutive models have been also developed to describe the strain rate dependence of thermoplastics (Regrain et al. (2009), Drozdov (2009), Drozdov et al. (2013), Vecchio et al. (2014) and Balieu et al. (2013)). To better reproduce the rate dependence in both elastic and inelastic deformation, coupled viscoelastic–viscoplastic (VE–VP) models have been proposed (Khan and Zhang (2001), Ayoub et al. (2010) and Miled et al. (2011)), these models can reproduce in an acceptable way the response of polymers.

Actually, the problem of structures subjected to rapidly oscillatory loading exhibits multiple temporal and spatial scales. It is a multiscale phenomenon in space (due to the presence of heterogeneities in the microstructure of the material) and time (because the load period could be in the order of seconds whereas the component life may span years).

The numerical solution process for complex, time-dependent non-linear problems requires, if one uses classical finite element (FE) codes, a computation time which turns out to be prohibitive. Thus new reliable and efficient strategies taking into account the multi-scale aspects in space and time are needed. In this paper we will consider only the case of solids made of homogeneous viscoelastic–viscoplastic (VE–VP) materials, subjected to large numbers of cycles. We will focus only on the two-scale time phenomena.

Relatively few works have been devoted to multiple time-scale phenomena, it is one of the areas in which there has been relatively little research and documentation. Smolinski et al. (1996) and Combescure and Gravouil (2002), introduced the so-called multi-time-step methods which allow to take into account different temporal discretizations in separate regions of the structure. This approach is used when a limited area of space requires a fine temporal solution or in the case of multiphysics problems for which different physical equations do not involve the same time scales.

However, treating problems at a very local scale with the technique presented above remains very costly. A method called variational multiscale method in time, was developed by Hughes and Stewart (1996) and Bottasso (2002). This approach is based on variational formulations in time and follows the same principles as those for the spatial aspects, where the basis functions form a partition of unity of the studied interval.

None of these strategies involves a true time-homogenization technique. Such techniques seem to have been introduced only for the case of cyclic loadings. There are two techniques that provide a temporal description suited to this type of problems. The first one is a strategy based on the so-called LATIN method developed by Cognard and Ladevèze (1993), which proposes a particular representation of the variables on two time scales: it is a temporal FE method. This method is known to be non incremental, i.e. at each iteration it generates an approximation of the solution on the entire time interval. The second technique is called two-scale time homogenization, which is a direct extension of the asymptotic spatial homogenization. It was developed by Guennouni (1988) for elasto-viscoplastic (EVP) homogeneous solids. It is based on asymptotic expansions in time of all the unknown fields and leads to a homogenized time behavior. This technique was also used in the case of cyclic loadings by Yu and Fish for thermo-viscoelastic composites (Yu and Fish (2001)), and for homogeneous materials following the Maxwell viscoelastic model and the power-law viscoplastic model (Yu and Fish (2002)). It was also used by Aubry and Puel to predict the long-term behavior of elastoplastic materials subjected to two-frequency periodic loads (Aubry and Puel (2010a, b)). The multiscale modeling was applied by Devulder et al. (2010) to fatigue damage evolution in cortical bone and Manchiraju et al. (2007, 2008), and Chakraborty et al. (2011) to study fatigue response of Ti alloys.

In this paper, we extend the two-scale time homogenization theory proposed by Guennouni (1988) from EVP materials to a constitutive model more suitable for thermoplastic polymers. The constitutive VE–VP model couples VE (with arbitrary Prony series for time-dependent shear and bulk moduli) and VP (with arbitrary isotropic hardening and general Perzyna-type VP functions).

An earlier version was published by the authors in (Haouala and Doghri (2013)). However, the present work presents the following important novelties with respect to the previous paper. First, the VE–VP model is developed within a thermodynamic framework from which state and evolution laws are derived (Section 2 and Appendix A). Second, computational fully implicit time integration algorithms are proposed (Section 4). Third, new numerical simulations are presented in Section 5 including multiaxial ones and a comparison with experimental data.

The paper has the following outline. In Section 2, a coupled VE–VP constitutive model which satisfies the Clausius-Duhem inequality is formulated within the framework of small strain theory and isothermal process. In Section 3, a time homogenization scheme for coupled VE–VP solids is presented. In Section 4, the computational algorithm is detailed and studied. In Section 5, the time homogenization approach is verified against reference full-time solution for several loading cases and compared to experimental results in one case.

Throughout the paper, the following notations and conventions are used. Boldface symbols designate second- or fourth-rank tensors as indicated by the context. The different products are expressed as:

$$\mathbf{a} : \mathbf{b} = a_{ij} b_{ji}, \quad (\mathbf{C} : \mathbf{D})_{ijkl} = C_{ijmn} D_{nmkl}, \quad (\mathbf{a} \otimes \mathbf{b})_{ijkl} = a_{ij} b_{kl},$$

where summation over a repeated index is supposed. The symbols 1 and $\mathbf{I}$ stand for the second- and symmetric fourth-order identity tensors, respectively. Finally, deviatoric and volumetric operators $\mathbf{I}^{dev}$ and $\mathbf{I}^{vol}$ are given by: