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New three-dimensional strain-rate potentials for isotropic porous metals: Role of the plastic flow of the matrix

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A B S T R A C T

At present, modeling of the plastic response of porous solids is done using stress-based plastic potentials. To gain understanding of the combined effects of all invariants for general three-dimensional loadings, a strain-rate based approach appears more appropriate. In this paper, for the first time strain rate-based potentials for porous solids with Tresca and von Mises, matrices are obtained. The dilatational response is investigated for general 3-D conditions for both compressive and tensile states using rigorous upscaling methods. It is demonstrated that the presence of voids induces dependence on all invariants, the noteworthy result being the key role played by the plastic flow of the matrix on the dilatational response. If the matrix obeys the von Mises criterion, the shape of the cross-sections of the porous solid with the octahedral plane deviates slightly from a circle, and changes very little as the absolute value of the mean strain rate increases. However, if the matrix behavior is described by Tresca's criterion, the shape of the cross-sections evolves from a regular hexagon to a smooth triangle with rounded corners. Furthermore, it is revealed that the couplings between invariants are very specific and depend strongly on the particularities of the plastic flow of the matrix.

1. Introduction

Using the plastic work equivalence principle, Ziegler (1977) has shown that a strain-rate potential can be associated to any convex stress-potential. Thus, to describe plastic flow a strain-rate potential (SRP) can be used instead of a stress-based potential. Such an approach is generally adopted in crystal plasticity because it is much easier to calculate numerically a crystallographic strain-rate potential than to compute a crystallographic yield surface (e.g. see Van Bael and Van Houtte, 2004).

Analytic expressions of macroscopic strain-rate potentials associated to stress-based potentials are known only for classical isotropic yield criteria such as Mises, Tresca, or Drucker–Prager (see Salencon, 1983), the orthotropic Hill (1948) criterion (see Hill, 1987), the orthotropic criterion of Cazacu et al. (2006) (see Cazacu et al. 2010; Yoon et al., 2011). If the stress-based potentials are non-quadratic obtaining an analytic expression for the exact dual is very challenging, if not impossible. Barlat and co-workers have proposed several analytic non-quadratic anisotropic strain rate potentials (e.g. Barlat et al., 2003; Kim et al., 2007). Although none of these strain rate potentials are strictly dual (conjugate) of the respective stress potentials, it was shown that these strain rate formulations lead to a description of the plastic anisotropy of FCC

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metals of comparable accuracy to that obtained using the stress potentials. It is generally agreed that strain-rate potentials are more suitable for process design involving the solution of inverse problems (see Barlat et al., 1993; Chung et al., 1997).

It is to be noted that all the strain-rate based potentials mentioned above apply only to fully-dense materials for which the plastic flow is incompressible. However, all engineering materials contain defects (cracks, voids). Based on micromechanical considerations, Rice and Tracey (1969) and Gurson (1977) have demonstrated that the presence of voids induces a dependence of the mechanical response on the mean stress and as such the plastic flow is accompanied by volume changes. Stress-based potentials have been developed to capture the characteristics of the plastic flow of porous metallic materials. Most of these models are based on the hypothesis that the matrix (void-free material) obeys von Mises yield criterion. This is the case, for example, of the classical Gurson (1977) model and its various extensions proposed by Tvergaard (1981), Gologanu (1997), Tvergaard and Nielsen (2010), Lecarme et al. (2011), and among others. It is to be noted that in all these Gurson-type models as well as in recent phenomenological models (e.g. Stoughton and Yoon, 2011; Yoon et al. 2013, etc.) the effects of the mean stress and shear stresses are decoupled. However, finite-element (FE) cell model calculations as well as very recent full-field calculations of the yield surface of voided polycrystals deforming by slip have shown a very specific dependence of yielding with the signs of the mean stress and the third-invariant of the stress deviator, J_3^Σ. Specifically, for tensile loadings the response corresponding to $J_3^\Sigma \geqslant 0$ is softer than that corresponding to $J_3^\Sigma \leqslant 0$ while for compressive loadings, the reverse occurs (e.g. Richelsen and Tvergaard, 1994; Cazacu and Stewart, 2009; Lebensohn and Cazacu, 2012). For axisymmetric loadings, using micromechanical considerations Cazacu et al. (2013, 2014) developed analytical yield criteria for porous materials with von Mises and Tresca matrix, respectively. These stress-based potentials account for the combined effects of the sign of the mean stress and of the third-invariant of the stress deviator on the dilatational response. Most importantly, it was explained the role of the third-invariant on void growth and void collapse (see Alves et al., 2014). However, to gain understanding of the combined effects of all invariants for general three-dimensional loadings, a strain-rate based approach appears more appropriate.

The need for models that can explain and better predict damage accumulation and its influence on the plastic response of engineering materials was evidenced by recent experimental studies where the mechanical response for full three-dimensional loadings of both polycrystalline metallic materials (e.g. Barsoum and Faleskog, 2007; Maire et al., 2011; Khan and Liu, 2012, etc.) and microcellular metallic materials (or metallic "foams" (e.g. Combaz et al., 2011)). In particular, Combaz et al. (2011) reported data on aluminum foams for multi-axial loading paths corresponding to different values of J_3^Σ for both tensile and compressive loadings, showing that the yield locus is centro-symmetric, its shape in the octahedral plane being triangular.

In this paper, for the first time three-dimensional (3-D) strain-rate potentials for porous solids with von Mises and Tresca matrix are obtained and their properties investigated for both compressive and tensile loadings. The structure of the paper is as follows. We begin with a brief presentation of the modeling framework (Section 2). The strain-rate potentials for porous von Mises and Tresca solids are given in Section 3. To fully assess the properties of the respective potentials, the shape of the cross-sections with the octahedral plane are analyzed. For a porous solid with von Mises matrix, the noteworthy result is that the shape of the cross-section changes little with the mean strain rate and the most pronounced influence of the third-invariant occurs for axisymmetric states.

However, if the plastic flow in the matrix is governed by Tresca's criterion, there is a very strong coupling between all invariants. New features of the dilatational response are revealed. Irrespective of the level of porosity, the shape of the cross-section in the octahedral plane changes from a regular hexagon to a rounded triangle, the surface closing on the hydrostatic axis. Furthermore, it is shown that the level of porosity in the material strongly influences the couplings between the invariants. In Section 4, we revisit some aspects of Gurson's treatment. More specifically, the implications of the approximations made by Gurson (1975, 1977) and the properties of the strain-rate potential, which is the exact conjugate of its classical stress-based potential (Gurson, 1977)) are discussed in relation to the exact strain-rate potentials for porous solids with Tresca and von Mises matrix presented in this study (Section 4). The main findings of this paper are summarized in Section 5.

Regarding notations, vector and tensors are denoted by boldface characters. If **A** and **B** are second-order tensors, the contracted tensor product between such tensors is defined as: $\mathbf{A} : \mathbf{B} = A_{ij}B_{ij}$ $i, j = 1, \ldots, 3$.

2. Modeling framework

2.1. Strain-rate potentials for fully-dense metallic materials

Generally, the onset of plastic flow is described by specifying a convex yield function, $\varphi(\boldsymbol{\sigma})$, in the stress space and the associated flow rule

$$\mathbf{d} = \dot{\lambda}\frac{\partial \varphi}{\partial \boldsymbol{\sigma}}, \tag{1}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\mathbf{d}$ denotes the plastic strain rate tensor and $\dot{\lambda} \geqslant 0$ stands for the plastic multiplier. The yield surface is defined as: $\varphi(\boldsymbol{\sigma}) = \sigma_T$, where σ_T is the uniaxial yield in tension. The dual potential of the stress potential $\varphi(\boldsymbol{\sigma})$ is defined (see Ziegler, 1977; Hill, 1987) as: