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Thermal performance and optimization of hyperbolic annular fins under dehumidifying operating conditions – analytical and numerical solutions

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ABSTRACT

The thermal performance of hyperbolic profile annular fins subjected to dehumidifying operating conditions is studied. An analytical solution for completely wet fin is derived using an approximate linear temperature–humidity relationship. A numerical solution using actual psychrometric relationship for completely and partially wet operating conditions is then obtained to account for the actual temperature–humidity ratio psychrometric relationship under both partially and fully wet operating conditions. An excellent agreement is observed between analytical and numerical solutions for completely wet fin. The fin optimization is presented based on the analytical solution of completely wet fin. Finally, a finite element formulation is used for studying the two-dimensional effects of orthotropic thermal conductivity on the thermal performance of fin under partially and fully wet operating conditions.

Performance thermique et optimisation d'ailettes annulaires hyperboliques sous conditions de fonctionnement déshumidifiant – solutions analytique et numérique

Mots clés : Ailette annulaire ; Profil hyperbolique ; Solution analytique ; Optimisation d'ailette ; Analyse d'élément fini ; Conductivité thermique orthotrope ; Transfert de masse ; Ailette partiellement humide

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Nomenclature

B	Parameter defined in Eq. (10) [°C]
Bi	Biot number
Co	constant defined in Eq. (33) [kg_w/kg_a]
c_p	specific heat of incoming moist air stream [$J\ kg^{-1}\ K^{-1}$]
h	convective heat transfer coefficient [$W\ m^{-2}\ K^{-1}$]
h_D	mass transfer coefficient [$kg\ m^{-2}\ s^{-1}$]
h'	equivalent heat transfer coefficient defined by Eq. (69) [$W\ m^{-2}\ K^{-1}$]
i_{fg}	latent heat of evaporation for water [$J\ kg^{-1}$]
k	thermal conductivity [$W\ m^{-1}\ K^{-1}$]
L	fin length [m]
Le	Lewis number
M	parameter defined in Eq. (21) [$m^{-3/2}$]
M_0	parameter defined in Eq. (20) [$m^{-3/2}$]
m_0	parameter defined in Eq. (20) [m^{-1}]
P_{atm}	atmospheric pressure [Pa]
Q	dimensionless heat flow rate
Q^*	heat flow rate [W]
q	heat flux [$W\ m^{-2}$]
q_b	specified heat flux [$W\ m^{-2}$]
RH	relative humidity of air
r^*	fin radius [m]
r	dimensionless fin radius
R	annular fin radius ratio
T	temperature [°C]
T_a'	equivalent ambient temperature defined by Eq. (70) [°C]
t^*	fin thickness [m]
t	dimensionless fin thickness

Greek symbols

η	efficiency
ψ	fin aspect ratio defined after Eq. (37)
θ	dimensionless temperature
τ	temperature difference defined by Eq. (16)
τ_p	temperature value defined by Eq. (26)
Ω	parameter defined in Eq. set (32)
ω	humidity ratio of air [$kg_w\ kg_a^{-1}$]

Finite element matrices and vectors

$\{\mathbf{f}\}$	element load vector
$[\mathbf{k}]$	element stiffness matrix
$[\mathbf{B}]$	flux-temperature matrix
$[\mathbf{D}]$	material property matrix
$[\mathbf{N}]$	shape function matrix
$\{\mathbf{T}\}$	nodal temperature vector

Subscripts and superscripts

a	ambient
b	base
h	convection
q	conduction
r	radial
t	tip
z	lateral
ζ	border between wet and dry region of a partially wet fin

1. Introduction

Extended surfaces are widely used to enhance the rate of heat transfer between a solid and a surrounding fluid. In refrigeration and air conditioning equipment, if the fin surface temperature is lower than the dew-point temperature of incoming moist air, then the condensation of water vapor occurs on the fin surface such that heat and mass transfer occurs simultaneously.

The major variables that influence the heat and mass transfer include, fin geometry, fin material and operating conditions. The fin geometries that are commonly used in these heat transfer devices may be classified as (a) spines or pin fins (b) longitudinal or straight fins, and (c) radial or annular fins, (Kraus et al., 2001). The objective of a variable profile fins is to provide high heat transfer capability for a given additional weight of the fin or to provide a minimum weight for the required amount of heat to be dissipated from the finned surface. The profiles are classified as (a) rectangular, (b) triangular or trapezoidal, (c) convex parabolic and (d) concave parabolic (Yovanovich, 2004). It is stated that among the whole family of annular fins of tapered cross section, the annular fin of hyperbolic profile is the foremost fin shape for usage in tubes of high performance heat exchange devices (Campo and Cui, 2008).

The performance of a fin is well described by its efficiency, defined as:

$$\eta = \frac{Q^*}{Q^*_{max}} \tag{1}$$

where Q is heat transfer rate through the fin and Q_{max} is the maximum possible heat transfer rate from the fin if the entire fin surface is at the prime surface temperature and humidity ratio.

High efficiency fins are desirable for effective heat transfer. For the case of dry operating conditions, the rate of sensible heat loss can be increased by using force convection e.g. by using a fan. However, for a given fin material and geometry, the maximum value of convective heat transfer under dry operating conditions is governed by the dimensionless parameter called Biot number. For a circular cross-section pin fin it is given by:

$$\mathrm{Bi} = \frac{hr}{k} \tag{2}$$

h is the convective heat transfer coefficient, k is the thermal conductivity and r is the fin radius. For metallic fin material, a fin has high efficiency (>90%) values only in very low Biot number range $\mathrm{Bi} \ll 1$. For a given fin material, this condition implies that low h values must be used to have high efficiency slender pin fins. This issue can be addressed by using a fin material with orthotropic thermal conductivity; i.e. a pin fin with different thermal conductivities k_r and k_z in radial and