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Theoretical and numerical modeling of shape memory alloys accounting for multiple phase transformations and martensite reorientation

F. Auricchio [a], E. Bonetti [b], G. Scalet [c,*], F. Ubertini [c]

[a] Dipartimento di Ingegneria Civile e Architettura, Università di Pavia, Via Ferrata 3, 27100 Pavia, Italy
[b] Dipartimento di Matematica, Università di Pavia, Via Ferrata 1, 27100 Pavia, Italy
[c] Dipartimento di Ingegneria Civile, Chimica, Ambientale e dei Materiali, Università di Bologna, Viale Risorgimento 2, 40136 Bologna, Italy

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ABSTRACT

The present paper develops a refined and general three-dimensional phenomenological constitutive model for shape memory alloys (SMAs), along the lines of what recently proposed by Auricchio and Bonetti (2013) in a more theoretical context. Such an improved model takes into account several physical phenomena, as martensite reorientation and different kinetics between forward/reverse phase transformations, including also smooth thermo-mechanical response, low-stress phase transformations as well as transformation-dependent elastic properties. The model is treated numerically through an effective and efficient procedure, consisting in the replacement of the classical set of Kuhn–Tucker inequality conditions by the so-called Fischer–Burmeister complementarity function. Numerical predictions are compared with experimental results and the finite element analysis of a SMA-based real device is described to assess the reliability of the proposed model as well as the effectiveness of its numerical counterpart.

1. Introduction

Smart materials exhibit special properties that make them an attractive choice for industrial applications in many branches of engineering. Among different types of smart materials, shape memory alloys (SMAs) have unique features known as pseudo-elasticity (PE), one-way and two-way shape memory effects (SMEs) (Duerig et al., 1990; Otsuka and Wayman, 1998). Such unusual effects are exploited in a large variety of interesting applications. The most successful commercial examples are in the biomedical area, e.g., endo-prosthesis, orthodontic archwires, cardiovascular stents (Wu et al., 2007; Auricchio et al., 2010a; Azaouzi et al., 2013), as well as in the robotic and automotive areas, e.g., positioning for mirror seats, actuators, micro-grippers (Auricchio et al., 2009a, 2010b; Williams and Elahina, 2008; Huang, 1998).

SMA features are the consequence of reversible martensitic phase transformations (PTs) between a high symmetric austenitic phase and a low symmetric martensitic phase. Austenite is a solid phase, present at high temperature, which transforms into different possible martensitic variants by means of a lattice shearing mechanism. In thermal-induced transformations under zero stress, multi-direction martensite variants compensate each other and arrange themselves in a self accommodating manner through twinning, with no observable macroscopic shape change. In stress-induced

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[*] Corresponding author. Tel.: +39 051 2093374.
E-mail address: giulia.scalet2@unibo.it (G. Scalet).

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transformations, starting from a martensitic specimen, the application of a loading induces a detwinning process of the martensitic variants, leading to the presence of a single-variant (Duerig et al., 1990); upon unloading, a large residual strain remains, which can be recovered by heating. This phenomenon is referred to as SME. On the other hand, when a stress is applied to an austenitic specimen, at high temperature, a transformation from austenite to single-variant martensite occurs; upon unloading, the strain attained during loading is recovered. This process is referred to as PE (Otsuka and Wayman, 1998).

Such functional material properties motivate researchers to formulate constitutive models able to catch the interesting behavior of SMAs and to develop robust computational tools for practical purposes. In the following, we focus on both the constitutive and numerical modeling of SMAs by briefly reviewing some approaches available from the literature and by carefully describing our motivations and proposed improvements.

1.1. Constitutive modeling: state of art and proposed improvements

In the past three decades SMAs have been deeply investigated from the point of view of modeling, analysis, and computation with the focus on a variety of aspects, such as, for instance, stress- and temperature-induced transformations, martensite reorientation or cyclic effects.

In terms of modeling, there have been several attempts to properly reproduce SMA material features. The resulting models can be categorized as either micro, micro–macro or macro. For an overview, see Khandelwal and Buravalla (2009), Lagoudas et al. (2006) and Patoor et al. (2006).

In the following, we focus on phenomenological macro-modeling approaches which appear to be a powerful tool for the direct simulation of SMA applications, thanks to their simple numerical implementation and reduced time-consuming calculations, compared to micro-mechanical approaches. In particular, the present research is devoted to the aim of finding a flexible and accurate three-dimensional phenomenological model for a reliable description of SMA-based real devices behavior.

In the phenomenological framework, an appropriate set of internal variables has to be chosen to represent at least a scalar and a directional information (Luig and Bruhns, 2008). Physical motivations usually lead to the introduction of a martensite volume fraction and of a tensorial variable describing martensitic inelastic deformation processes (Arghavani et al., 2010; Luig and Bruhns, 2008; Peultier et al., 2006; Saleeb et al., 2011). Such a simplified description is motivated by the aim to obtain fast and efficient models with a low number of fitting parameters.

A set of only scalar variables is, in fact, not adequate due to the loss of explicit directional information. For instance, the model by Frémond (2002) describes SMA behavior in terms of austenite and two martensite variants and assumes the transformation strain direction to be known, although experimental studies showed that variant reorientation can be considered as a main phenomenon in SMA non-proportional loadings (Bouvet et al., 2002; Grabe and Bruhns, 2009; Lim and McDowell, 1999; Sittner et al., 1995; Sun and Li, 2002; Helm and Haupt, 2003).

On the other hand, models with only tensorial internal variables, by explicitly including simple directional information, seem to be more successful, but present some limitations since scalar and directional informations are tightly interconnected, possibly leading to limited or constrained modeling approaches. As an example, the model by Souza et al. (1998), then investigated by Auricchio and Petrini (2004a,b), introduces the transformation strain tensor as an internal variable and presents a simple and robust algorithm, widely used for implementation within finite element (FE) codes. On the contrary, it is not able to capture PTs for low levels of stress, as required often by industrial applications (Auricchio et al., 2009a), and does not include some secondary effects that may turn out to be relevant in practical cases (Thamburaja and Anand, 2001).

Numerous analyses of existing models and their comparison to experimental results have shown that current SMA constitutive models have reached a high level of sophistication. Several authors extended, in fact, such simplified phenomenological descriptions by using additional variables as volume fraction of twinned/detwinned martensites (Lexcellent et al., 2000; Panico and Brinson, 2007; Popov and Lagoudas, 2007), twins accommodation strain (Chemisky et al., 2011), viscoplasticity (Chemisky et al., 2014), thermo-mechanical coupling (Morin et al., 2011a,b; Zaki and Moumni, 2007a,b) or plastic strain (Auricchio et al., 2007; Hartl et al., 2010; Zaki et al., 2010; Saint-Sulpice et al., 2009; Peng et al., 2012). The recent and innovative work by Sedlák et al. (2012) formulates a new dissipation function to simulate non-proportional loadings and includes anisotropic behavior of textured SMAs as well as the thermo-mechanical response due to austenite-R-phase transformation. Panoskaltsis et al. (2004) developed a three-dimensional thermo-mechanical constitutive model based on generalized plasticity theory in the small deformation regime, and Panoskaltsis et al. (2011a,b) within finite strains and rotations.

However, the most capable models usually achieve accuracy at the cost of complexity, since they consider multiple and simultaneous processes (Popov and Lagoudas, 2007; Chemisky et al., 2011) or require costly calibrations of a high number of model parameters (Saleeb et al., 2011, 2013a).

Starting from the reviewed literature about constitutive modeling, the present paper is motivated by the necessity of developing constitutive models that can predict the complex thermo-mechanical behavior of SMAs and that can also be implemented numerically. Such models have to accurately capture material response not only during classical PE and SME loading paths, but also during loading paths involving the co-existence of all the three material phases, i.e., austenite, multiple- and single-variant martensite. Moreover, model material parameters have to be derived from a simple physical