Materials Science and Engineering C 60 (2016) 163–183

Contents lists available at ScienceDirect

Materials Science and Engineering C

journal homepage: www.elsevier.com/locate/msec

# Mechanical properties of regular porous biomaterials made from truncated cube repeating unit cells: Analytical solutions and computational models

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ARTICLE INFO

*Article history:*
Received 30 March 2015
Received in revised form 16 October 2015
Accepted 3 November 2015
Available online 10 November 2015

*Keywords:*
Porous biomaterials
Mechanical behavior
Analytical
Lattice structure

ABSTRACT

Additive manufacturing (AM) has enabled fabrication of open-cell porous biomaterials based on repeating unit cells. The micro-architecture of the porous biomaterials and, thus, their physical properties could then be precisely controlled. Due to their many favorable properties, porous biomaterials manufactured using AM are considered as promising candidates for bone substitution as well as for several other applications in orthopedic surgery. The mechanical properties of such porous structures including static and fatigue properties are shown to be strongly dependent on the type of the repeating unit cell based on which the porous biomaterial is built. In this paper, we study the mechanical properties of porous biomaterials made from a relatively new unit cell, namely truncated cube. We present analytical solutions that relate the dimensions of the repeating unit cell to the elastic modulus, Poisson's ratio, yield stress, and buckling load of those porous structures. We also performed finite element modeling to predict the mechanical properties of the porous structures. The analytical solution and computational results were found to be in agreement with each other. The mechanical properties estimated using both the analytical and computational techniques were somewhat higher than the experimental data reported in one of our recent studies on selective laser melted Ti–6Al–4V porous biomaterials. In addition to porosity, the elastic modulus and Poisson's ratio of the porous structures were found to be strongly dependent on the ratio of the length of the inclined struts to that of the uninclined (i.e. vertical or horizontal) struts, $\alpha$, in the truncated cube unit cell. The geometry of the truncated cube unit cell approaches the octahedral and cube unit cells when $\alpha$ respectively approaches zero and infinity. Consistent with those geometrical observations, the analytical solutions presented in this study approached those of the octahedral and cube unit cells when $\alpha$ approached respectively 0 and infinity.

## 1. Introduction

Recent advances in additive manufacturing techniques have enabled fabrication of regular porous biomaterials with precisely defined micro-architectures [1,2]. The micro-architecture of porous biomaterials is shown to determine their physical properties including static mechanical properties [3,4], fatigue resistance [5], and permeability [6]. In addition to physical properties, the geometrical features of porous biomaterials at the small scale such as the curvature of the pores and its sign, are shown to regulate the biological response to porous biomaterials [7]. In particular, it has been observed that the rate of tissue regeneration is dependent on the above-mentioned features [7–10].

Given the importance of micro-architecture in determining the physical and biological properties of porous biomaterials, it is important to systematically study the relationship between the micro-architecture of porous materials and their physical and biological properties. The role of the geometry of the repeating unit cell used for additive manufacturing of regular porous biomaterials on the mechanical properties of these structures must be studied within this context. The mechanical properties of porous biomaterials that are used for bone substitution have been therefore receiving increasing attention recently. That is partly due to the fact that matching the original mechanical properties of the porous biomaterials to those of bone is now possible by simply choosing the right type of unit cell and by adjusting the dimensions of the chosen unit cell. In this way, one could optimally distribute the mechanical properties of the biomaterial within an orthopaedic implant so as to minimize the effects of the stress-shielding phenomenon and to decrease the undesired consequences that are associated with stress-shielding (and are well described in the literature, see e.g. [11–13]).

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http://dx.doi.org/10.1016/j.msec.2015.11.001
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Moreover, the pores of porous biomaterials could be used as reservoirs for delivering growth factors [14] and potentially other types of biomolecules. The huge surface area of the porous structures is another opportunity that combined with proper types of surface treatment and coating could contribute towards improving the bio-activity of the biomaterials and may ultimately result in improved bone regeneration performance and osseointegration [14].

The mechanical properties of porous structures based on various types of unit cells have been studied before [2,15–31]. Truncated cube lattice structure is a relatively new morphology whose mechanical properties have not been studied extensively. The crushing behavior of closed-cell foams based on the truncated cube morphology, as a good representative of traditional foams, has been investigated numerically by a number of researchers [32–36]. Elastic modulus, Poisson's ratio, and yield stress of open- and closed-cell truncated cube periodic lattices have also been investigated numerically in [34]. While the number of numerical investigations of porous structures based on the truncated cube unit cell is inadequate for closed-cell structure and limited to only one study for open-cell structures (in which the properties are obtained for only one relative density [34]) no analytical solution has been presented for predicting the mechanical properties of this structure with open or closed cells.

Analytical solutions that relate the micro-architecture of porous biomaterials to their mechanical properties are useful from different viewpoints. First, they could help in understanding the deformation and failure mechanisms of those biomaterials. Second, analytical solutions could be used for validation of computational results. Finally, they may serve as a replacement for computational models when running patient-specific optimization algorithms that require many estimations of the mechanical properties of porous biomaterials.

In this paper, analytical solutions are derived to estimate the elastic modulus, Poisson's ratio, yield stress, and buckling load of open-cell porous biomaterials made from the truncated cube unit cell. Finite element models are developed to compare the analytical solutions with computational results. Moreover, experimental data from one of our previous studies is used for validation.

## 2. Methodology

The truncated cube (Fig. 1), also known as truncated hexahedron, is one of the limited number of unit cells which can produce a lattice structure after being repeated in space. In this study, the considered unit-cell is open-cell and the inclined and uninclined struts can have equal ($m=l$) or unequal ($m\neq l$) lengths (Fig. 1). The ratio of the length of the inclined struts to that of the uninclined (i.e. vertical or horizontal) struts are denoted $\alpha$ here and its effect on the obtained elastic properties will be investigated. The three sub-sections of this section cover the derivation of the analytical solutions, the description of the finite element models used for estimation of the mechanical properties of the porous structures, and a description of the experimental data used here.

### 2.1. Analytical solution

### 2.1.1. Relative density

Relative density (or apparent density) is defined as the ratio of the density of a porous structure to the density of the solid material that it is made of. In this study, analytical relationships are presented for mechanical properties of truncated cube lattice structures having struts with three different cross-section types, namely circle, square, and equilateral triangle. Therefore, the relative density relationships are also obtained for the three noted cross-section types. Each truncated cube unit cell consists of 24 inclined edges with length $l$ (each shared by another adjacent unit cell) and 12 uninclined (i.e. vertical or horizontal) edges with length $m=2\alpha l$ (each shared by three adjacent unit cells), see Fig. 1. Each truncated cube unit cell therefore occupies a volume of $V_{uc}=(\sqrt{2}l+m)^3=(2\alpha+\sqrt{2})^3 l^3$. The relative density of this structure can be obtained by dividing the volume occupied by the material to the total volume of the unit cell. For the circular cross-section, we have

$$\mu=\frac{V_{struts}}{V_{uc}}=\frac{\frac{24}{2}\{\pi r^2 l\}+\frac{12}{4}\{2\pi r^2 \alpha l\}}{\left(2\alpha+\sqrt{2}\right)^3 l^3}=\frac{\pi(12+6\alpha)}{\left(2\alpha+\sqrt{2}\right)^3}\left(\frac{r}{l}\right)^2. \tag{1}$$

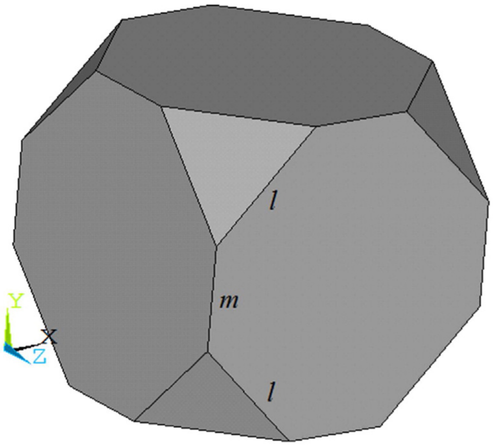

**Fig. 1.** One of the unit cells constructing a truncated cube lattice structure.