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A stress-based variational model for ductile porous materials

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ABSTRACT

The main objective of this paper is to formulate a very new statically-based model of ductile porous materials having a von Mises matrix. In contrast to the Gurson's well known kinematical approach applied to a hollow sphere, the proposed study proceeds by means of a statical limit analysis procedure. Its development and implementation require the choice of an appropriate trial stress field. The starting point is Hill's variational principle for rigid plastic matrix. The use of a lagrangian multiplier allows to satisfy the plastic criterion in an average sense. The proposed trial stress field, complying with internal equilibrium, is composed of an heterogeneous part (exact solution for the stress field in the hollow sphere under pure hydrostatic loading) to which is added a uniform deviatoric stress field. Owing to this choice, the stress vector conditions on the void boundary are relaxed. By solving the resulting Saddle point problem, we derive closed form formula which depends not only on the first and second invariant of the macroscopic stress tensor but also on the sign of the third invariant of the stress deviator. The obtained results are fully discussed and compared to existing models, available numerical data and to Finite Elements results obtained from cell calculation carried out during the present study. Finally, we provide for the new model the macroscopic flow rule as well as the porosity evolution equations which also show very original features. Some of these features are illustrated.

1. Introduction

To obtain macroscopic criterion of ductile porous materials, Gurson (1977) has proposed a kinematical limit analysis approach of a hollow sphere and a hollow cylinder having a von Mises solid matrix. This approach delivered an upper bound of the searched macroscopic criterion. Several extensions of the Gurson model have been further proposed in the literature. Owing to the observation that Gurson model appears too stiff when compared with finite element unit-cell computations, Tvergaard (1981) and then Tvergaard and Needleman (1984) (see also Tvergaard (1981) and Tvergaard (1990)) proposed a heuristic extension of the Gurson model, known as the GTN model. This extension, widely used in structural computations introduces three parameters, q_1, q_2 and q_3, which have to be determined. A proposition on the dependence of such parameters on porosity has been recently done by Fritzen et al. (2012) based on computations carried out on a representative elementary volume of the ductile porous medium.[1] Applications of ductile fracture models generally concern metallic materials (see for instance Gänser et al. (1998), Han et al. (2013), Khan and Liu (2012) and Khan and Liu (2012)). Further extensions of the Gurson model, probably the most important ones, include those accounting for void shape effects (Gologanu et al., 1997; Garajeu et al., 2000; Monchiet et al., 2007; Monchiet et al., 2013). Matrix plastic anisotropy was also treated for the first time by Benzerga and Besson (2001) in the case of spherical voids, the extension to spheroidal voids being made later by Monchiet et al.

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[1] This study has been extended in Fritzen et al. (2012) to porous materials with Green type matrix.

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(2006) and Monchiet et al. (2008) (see also Keralavarma and Benzerga (2008) and Keralavarma and Benzerga (2010)). Other recent works concern ductile porous metals with incompressible matrix exhibiting an asymmetry between tension and compression (see for instance Cazacu and Stewart (2009) and Revil-Baudard and Cazacu (2013)). For completeness, mention has to be made of other theoretical extensions taking into account the plastic compressibility of the matrix by considering a Mises–Schleicher or a Drucker–Prager matrix for applications to polymer and cohesive geomaterials (Jeong and Pan, 1995; Jeong, 2002; Lee and Oung, 2000; Zaïri et al., 2008; Canal et al., 2009; Guo et al., 2008). Finally, some very recent works deal with nanoporous materials (Huang and Li, 2005; Li and Huang, 2005; Zhang et al., 2010; Dormieux and Kondo, 2010; Dormieux and Kondo, 2013; Monchiet and Kondo, 2013). Others are devoted to porous materials reinforced by rigid inclusions (Garajeu and Suquet, 1997; Shen et al., 2012; He et al., 2013).

In Gurson's footsteps, all the above limit analysis-based models of ductile porous media are obtained by using kinematical approach which requires the choice of a suitable trial velocity field[2]. On the other hand, few works have been made to develop a theoretical dual stress based model. One may mention the pioneering study of Green (1972), even it has been phenomenologically inspired. A statical limit analysis attempt has been first done for ductile porous media by Sun and Wang (1989) (see also Sun and Wang (1995)) who developed a semi-analytical approach which aimed to deliver a lower bound criterion.[3] Despite the interest of the above approaches by Sun and Wang (1989) and by Landry and Chen (2011), the resulting criteria are in fact obtained by some fitting procedure based on numerical computations. Moreover, although the authors claim a more accurate formability prediction than by the Gurson criterion when compared to experiments, the accuracy of the above criteria (built upon analysis of a hollow sphere subjected to uniform stress boundary conditions) seems not satisfactory in view of rigorous numerical bounds recently obtained by Trillat and Pastor (2005) and Thoré et al. (2011) from statically-based numerical computations and optimization made on the hollow sphere. It clearly appears that a proper theoretical framework based on a statical limit analysis of porous media, and its implementation for the derivation of a macroscopic criterion, are still due. This is the main purpose of the present study.

From a more general point of view, it must be noted that, although the direct and accurate knowledge of the stress field is of great interest in plasticity due to the fact that the yield criterion is often expressed in terms of stress components, the main reason which probably explains the preference in past studies of the kinematical approach (leading to upper bounds) is technical: the dissipation function is non smooth but only for null plastic strains. As it is generally the case for limit analysis of microporous ductile materials, the reference cell (the hollow sphere in the present study) is completely plastified at limit state and the dissipation functional is smooth, differentiable with respect to the trial velocity field parameters. And as it is well known in duality theory, the more a functional is smooth, the more its dual one is non smooth. It is exactly what occurs in plasticity where the stress functional is much more difficult to manage due to its non smoothness concentrated in the satisfaction of the yield criterion.

The principal aim of this paper is to face this difficulty and to open a new way –alternative and complementary to Gurson like models– to build macroscopic yield criteria for ductile porous materials thanks to a stress model leading to a closed form expression of the macroscopic criterion. The developed approach also enters in the framework of limit analysis, well-known as a general method to determine the plastic limit state of structures under proportional loading. The variational formulation of the lower bound theorem is based on Hill's functional (Hill, 1950) (see also Nguyen Dang (1976) or chapter 6 in Save et al. (1997)) which is summarized and adapted to the homogenization problem by applying it to the hollow sphere model. The lower bound character of the results is guaranteed only if the trial stress field is statically and plastically admissible. This is the pitfall because, while the equilibrium equations are linear, the yield criterion is generally non linear, and then difficult to fulfill exactly. Of course, it could seem attractive to use linearised criteria such as Tresca or Mohr–Coulomb ones but they become non linear when expressed with respect to stress components in non principal axes. Still, in the framework of limit analysis, other linearisation were proposed but are not relevant because, to be accurate enough, they require in three dimensions consideration of too numerous linear facets. In the present paper, we follow an alternative approach inspired from numerical works by Nguyen Dang (1976) where the yield criterion is satisfied *'in the mean'* over each finite element. This concept has the following meaning: the yield condition is relaxed and we must expect to obtain only an approximation of the lower bound, sometimes qualified of quasi-lower bound. The idea seems to be relevant also for theoretical models because of the very high difficulty to obtain a closed analytical expression. The key idea is to satisfy only the equilibrium equations, relaxing the plastic criterion with Lagrange's multipliers. Moreover, the stress condition at the void boundary is also difficult to satisfy by simple trial stress fields capturing the shear effects that break the central symmetry; it will be also relaxed. A priori, the final picture could seem too rough but, although the trial stress field is rather simple with a strict number of field parameters able to fit the hydrostatic and deviatoric macro-stress components, the present approach provides a rather accurate model, as it will be shown. Indeed, the lower bound will be lost but, but by comparison to accurate numerical data, the interest and the validity of the new results will be demonstrated. Besides, a salient feature which will be shown for the derived model is that it predicts explicit dependence on all the three stress invariants. This particular point will be fully commented and analysed in relation with very recent results established by Cazacu et al. (2013) based on a kinematical limit analysis approach.

[2] Note that another class of ductile porous models, dealing with representative elementary volume, has been proposed in literature by using nonlinear homogenization techniques (see for instance Ponte-Castañeda (1991), Suquet (1995), Ponte-Castañeda and Suquet (1998), Barthélémy and Dormieux (2003), Danas et al. (2008) and Maghous et al. (2009)). These techniques are not discussed in the present paper.

[3] Note also the more recent study by Landry and Chen (2011) in which has been formulated a plane stress lower bound criterion for porous ductile sheet metals.