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Analytical solution for combined heat and mass transfer in laminar falling film absorption with uniform film velocity – Isothermal and adiabatic wall

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ABSTRACT

In the present study the Laplace transform is applied to the partial differential equations obtained from the differential energy and absorbate balances for the combined heat and mass transfer problem in laminar falling films with uniform film velocity.

By means of the inverse Laplace transform an analytical solution is provided for the isothermal as well as for the adiabatic wall boundary condition. Temperature and mass fraction profiles across the film as well as the evolution of the absorbed mass flux as a function of the flow length are presented for the adiabatic wall condition as well as for the isothermal wall with different wall temperatures. Furthermore, the influence of the Lewis number on the absorbed mass flux is discussed.

In addition, the present method allows to apply other wall boundary conditions than the isothermal or the adiabatic wall boundary, which will be addressed in a subsequent study.

Solution analytique pour le transfert combiné de chaleur et de masse dans l'absorption de film laminaire tombant avec vitesse de film uniforme – paroi isotherme et adiabatique

Mots clés : Absorption ; Solution analytique ; Film laminaire tombant ; Transfert de chaleur ; Transfert de masse ; Laplace

1. Introduction

By means of the Fourier method Grigor'eva and Nakoryakov (1977), Nakoryakov et al. (1997) and Nakoryakov and Grigor'eva (2010) presented an analytical solution for the combined heat and mass transfer in laminar falling film absorption with constant film velocity. Nevertheless, their solution did not match the inlet conditions for small dimensionless flow lengths ξ and wall temperatures, that are

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Nomenclature

Dimensionless numbers

Le	Lewis number ($Le = a\,D^{-1}$)
$\widetilde{St}$	modified Stefan number ($\widetilde{St} = c_s \cdot \Delta T \cdot \Delta h_{abs}^{-1} \cdot \Delta c^{-1}$)

Greek letters

α,β	eigenvalues
Δ	difference
δ	film thickness [m]
η	dimensionless film thickness
γ	dimensionless absorbate mass fraction
λ	thermal conductivity [$W \cdot m^{-1}K^{-1}$]
μ	dimensionless mass flux
ρ	density [$kg \cdot m^{-3}$]
Θ	dimensionsless temperature
ξ	normalized flow coordinate

Latin letters

a	thermal diffusivity [$m^2\,s^{-1}$]
A,B	constants [K]
c	mass fraction (absorbate) [$kg\,kg^{-1}$]
c	specific heat capacity [$kJ\,kg^{-1}\,K^{-1}$]
D	mass diffusivity [$m^2\,s^{-1}$]
h	specific enthalpy [$kJ\,kg^{-1}$]
i	imaginary unit
k	index
$\dot{m}$	mass flux [$kg \cdot m^{-2}s^{-1}$]
p	pressure [Pa]
T	temperature [K]
u	streamwise film velocity [$m \cdot s^{-1}$]
v	transverse film velocity [$m \cdot s^{-1}$]
v	specific volume [$m^3\,kg^{-1}$]
x	streamwise direction [m]
y	transverse direction to film flow [m]
z	complex Laplace variable

Sub-Superscripts/Symbols

0	inlet values
∞	asymptotic value for $\xi \to \infty$
$\overline{(\,)}$	mean value
abs	absorption
e	energy
eq	equilibrium (at corresponding inlet condition)
i	interface
i,j,k	index
m	mass
mA	mass fraction absorbate
s	solution
W	wall

different from the film inlet temperature (Nakoryakov and Grigor'eva, 2010). This mathematical problem originates in the domain restrictions of the tangent function which has been used to determine the eigenvalues by Nakoryakov and Grigor'eva (2010). By arranging these tangent functions to non-restricted sine and cosine functions (Meyer, 2014), more eigenvalues are found and the improved Fourier method matches the inlet condition for all boundary conditions even for small flow lengths ξ.

However, the orthogonality relation, which has been derived and used by Grigor'eva and Nakoryakov (1977), necessitates either the dimensionless wall temperature or the dimensionless wall temperature gradient to be zero. Thus, it is impossible to use any other wall boundary condition for the temperature than the isothermal or adiabatic wall, as the orthogonality relation is indispensable in order to apply the Fourier method.

For that reason in the present study the partial differential equations for energy and mass fraction are solved without any restrictions for the boundary conditions by means of the Laplace transform. However, the objective of this study is to introduce and validate the Laplace method by applying the isothermal and adiabatic wall only. Other boundary conditions will be presented in a subsequent study. The inverse Laplace transform presented by Baehr (1955) is applied in order to obtain the solutions to the combined heat and mass transfer problem in the real domain.

2. Film model

In order to illustrate the modelling assumptions, the differential absorbate and energy balance are applied to the film flow, leading to the usual partial differential equations for energy and mass fraction, which are the starting point for most of the analytical solutions.

Fig. 1 depicts the model of the film flowing down an isothermal, vertical wall for which, beside the adiabatic wall, the combined heat and mass transfer is considered.

In Fig. 2 an arbitrary infinitesimal volume element within the falling film is depicted and the streams marked by the arrows and labelled with j are fluxes of any arbitrary conservation quantity, e.g. mass or energy. Balancing this quantity j for steady state conditions leads to the following equation:

$$0 = [j(x) - j(x+dx)] \cdot dydz + [j(y) - j(y+dy)] \cdot dxdz. \tag{1}$$

The sign of the respective stream is determined by the direction of the arrow compared to the volume element as well as its direction compared to the direction of the coordinate. By means of a Taylor expansion neglecting the terms of an order $n>1$, it is possible to approximate the streams leaving the element as follows:

$$j(x+dx) = j(x) + \left.\frac{\partial j}{\partial x}\right|_x dx, \tag{2}$$

$$j(y+dy) = j(y) + \left.\frac{\partial j}{\partial y}\right|_y dy. \tag{3}$$

Introducing (2) and (3) to (1) only the derivatives of the conservation quantity remain:

$$0 = -\frac{\partial j}{\partial x} - \frac{\partial j}{\partial y}. \tag{4}$$

This is trivial for the steady state condition considered here. Every conservation quantity entering the control volume has to leave it, since a change in the respective quantity