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Improvement of the exact analytical solutions for combined heat and mass transfer problems obtained with the Fourier method

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ABSTRACT

Grigor'eva and Nakoryakov presented an analytical solution for combined heat and mass transfer in laminar falling films by means of the Fourier method. The obtained solutions exhibited mathematical instabilities for small flow length, such as oscillations in the mass fraction profile and a mismatch of the inlet temperature. Grigor'eva and Nakoryakov explained these instabilities with the inconsistency of the inlet and boundary conditions and therefore an additional short term solution was introduced.

In the present study the established tangent function, that is used to determine the eigenvalues within the Fourier method, is rearranged to a term without domain restrictions. Consequently, more eigenvalues are found, leading to a physical valid solution even for small flow lengths, matching the results of the short term solution perfectly.

Amélioration des solutions analytiques exactes pour des problèmes de transferts de chaleur et de masse combinées, obtenues grâce à la méthode de Fourier

Mots clés : Solution analytique ; Méthode de Fourier ; Transfert de chaleur ; Transfert de masse ; Solution exacte ; Absorption

1. Introduction

The combined heat and mass transfer in laminar falling film absorption is complex. Analytical solutions to this transfer problem with preferably realistic modeling assumptions are useful to understand the influence of the boundary conditions on e.g. the absorbed mass flux. In addition, analytical solutions, which cover the whole film flow, are suitable for

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Nomenclature

Dimensionless numbers

Le	Lewis number ($Le = a/D$)
$\widetilde{St}$	modified Stefan number ($\widetilde{St} = c_s \cdot \Delta T/(\Delta h_{abs} \cdot \Delta c)$)

Greek letters

α, β	eigenvalues
Δ	difference
δ	film thickness [m]
η	dimensionless film thickness
γ	dimensionless mass fraction
λ	thermal conductivity [W m^{-1} K^{-1}]
ρ	density [kg m^{-3}]
Θ	dimensionless temperature ($\Theta = (T - T_W)/(T_{eq} - T_0)$)
Θ^*	dimensionless temperature ($\Theta^* = (T - T_0)/(T_{eq} - T_0)$)
ξ	normalized flow coordinate

Latin letters

a	thermal diffusivity [m^2 s^{-1}]
A, B, C	constants
c	mass fraction (absorbate) [kg kg^{-1}]
c	specific heat capacity [kJ kg^{-1} K^{-1}]
D	mass diffusivity [m^2 s^{-1}]
i	imaginary unit
n	index
T	temperature [K]
u	streamwise film velocity [m s^{-1}]
x	streamwise direction [m]
y	transverse direction to film flow [m]
z	complex Laplace variable

Sub-/superscripts/symbols

0	inlet values, zero eigenvalue
$\overline{(\,)}$	mean value
abs	absorption
eq	equilibrium (at inlet condition)
i	interface
n	index
s	solution
st	short term
W	wall

comprehensive absorption heat pump simulation with reasonable computational effort as compared to numerical methods.

2. State of the art

By means of the Fourier method Grigor'eva and Nakoryakov (1977) presented the first analytical solution for combined heat and mass transfer in laminar falling film absorption with constant film velocity. For this solution Nakoryakov et al. (1997) reported oscillations in the mass fraction profile for small flow lengths. These mathematical instabilities were explained with the inconsistency of the inlet and boundary conditions. For that reason Nakoryakov et al. (1997) presented an additional short term solution in order to solve the problem for small flow lengths.

Grossman (1983) also applied the Fourier method to the partial differential equations for temperature and mass fraction. Instead of the uniform film velocity, Grossman applied a parabolic Nusselt film velocity profile. Grossman used infinite power series as eigenfunctions in order to solve the obtained non-linear ordinary differential equations. Consequently, the computational effort remarkably increases in comparison to the uniform film velocity as the problem's solution again forms infinite series of eigenfunctions. Moreover, the analytical solution of Grossman only converges for moderate and large values of the flow length and thus he needed an additional short term solution as well. Accordingly, Grossman (1983) also applied numerical methods to obtain the mass fraction and temperature profiles across the film for a parabolic film velocity profile.

The analytical models presented in literature subsequent to the complex analytical solutions of Nakoryakov et al. (1997) and Grossman (1983) obtained with the Fourier method were basically focused on the simplification of the problem. However, these simplifications restrict the respective range of validity.

Auracher et al. (2008) and Wohlfeil (2009) for instance presented a simplified model, solving the differential equations with first type boundary conditions at the interface. Wohlfeil (2009) assumed the temperature profile to be linear and described the mass transfer with the semi-infinite body model.

By means of the Laplace transform Meyer and Ziegler (2014) applied less restricted gradient expressions to the solving procedure of Wohlfeil (2009) and achieved only minor deviations to the solutions of Nakoryakov et al. (1997).

In the present study the solution of Nakoryakov et al. (1997) obtained with the Fourier method for the uniform velocity profile is improved, extending its range of validity to the whole film flow.

3. Film model

Fig. 1 depicts an absorbing falling film flowing down an isothermal, vertical wall with a constant mean film velocity $\overline{u}$.

Based on the differential energy and mass fraction balance for the laminar falling film with constant film velocity the following partial differential equations are obtained, allowing diffusive transport in transversal y and convective transport in streamwise direction x only.

$$\overline{u} \cdot \frac{\partial T}{\partial x} = a \cdot \frac{\partial^2 T}{\partial y^2}, \tag{1}$$

$$\overline{u} \cdot \frac{\partial c}{\partial x} = D \cdot \frac{\partial^2 c}{\partial y^2}. \tag{2}$$

The major simplifications of this approach are: