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The scaling exponent in the size effect of small scale plastic deformation

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ABSTRACT

Compression testing of micropillars (yield stress as a function of pillar diameter) has been interpreted in terms of very different scaling exponents for the size effect (smaller is stronger) in fcc and bcc metals and for ceramics, strongly correlated with the yield strengths of the materials. We show that the data is compatible with a single scaling exponent $x = 1$, and that the reported exponents in the range 0–1 express no more than the micropillar diameters and bulk strengths. The single scaling exponent implies a minimum strength which scales with size in the same way for all materials. This size effect is not fundamentally different in the presence of and in the absence of a strain gradient. It is interpreted in terms of the space available for dislocation source operation. The absence of any experimental data below this minimum strength is strong confirmation of this interpretation.

1. Introduction

The size effect (smaller is stronger) in micromechanical testing is now well established, in the flexure of thin metal foils (Stölken and Evans, 1998; Ehrler et al., 2008), in the torsion of thin metal wires (Fleck et al., 1994; Dunstan et al., 2009), in the nanoindentation of metals (Hou et al., 2008), ceramics (Zhu et al., 2008b) and semiconductors (Jayaweera et al., 2003), and in the uniaxial compression of micropillars fabricated from a great variety of materials from nickel (Uchic et al., 2004) to silicon (Moser et al., 2007). For recent reviews, see Kraft et al. (2010) and Zhu et al. (2008a). There are several competing or complementary theories that aim at accounting for the effect, either generally or, more modestly, in some particular circumstances. However, decisive experiments that can confirm or refute particular theories are scarce.

There is widespread agreement that the size effect should be expressible as a power law h^{-x} in some appropriate measure h of size, as in the Hall–Petch effects in which the yield strength and flow strength of simple polycrystalline metals vary as the inverse square root of the grain size (Hall, 1951; Petch, 1953). Matching experimental data to the power laws proposed by theory requires fitting the data to extract the scaling exponent x. This is often done by plotting large bodies of experimental data on log–log plots of strength against size and fitting to a straight line. In this way, it has been claimed that the scaling exponent x is different for different classes of materials with stronger materials displaying a less pronounced size effect with typical values of the exponent in the range 1–0.6 for fcc metals, around 0.5 or less for bcc metals, and as low as 0.2 or even zero for ceramics (Korte and Clegg, 2011). Yet no theory of the size effect predicts a scaling exponent varying in this way. Any theory that is based on the most fundamental physics of yield and plastic deformation by dislocations (the Orowan (1948) bowing stress, Brooks (1952) criterion, Matthews and Crawford (1970) critical thickness theory) immediately implies a scaling exponent of $x = 1$ to experimental accuracy (i.e., ignoring a term logarithmic in size). This is because the stress required to curve a dislocation to a radius r goes as r^{-1} (Orowan, 1948), and even more fundamentally because the plastic strain ε corresponding to a single dislocation of effective Burgers vector b in a finite space of dimension h is bh^{-1} (Brooks, 1952; Dunstan

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et al., 1991b). On the other hand, any theory that invokes the Hall–Petch effect and (or) analogy with Griffiths (1920) cracking theory and (or) Taylor (1934) forest hardening will naturally (for a variety of reasons; see Arzt 1998) give $x = ½$. We are not aware of any theory that can generally give other values.

In this paper we show that putting too much emphasis on what analytic expressions fit to the data can be misleading or indeed hide an underlying behavior. In Section 2 we show how large errors, of a factor of two or more in the scaling exponent x, can arise even with high-quality data sets. To avoid invidious comparisons, we use some of our own published data and fits. Many of the problems currently addressed in small-scale plasticity were resolved in semiconductor device research and this is described briefly in Section 3. Then in Section 4 we show the similarity of a large body of literature data in other materials and other configurations (particularly pillar compression) to the semiconductor plastic relaxation behavior and scaling exponent.

In Section 5, we use these results to present a different interpretation of the size effect by considering the elastic strain required to operate a dislocation source in a confined volume. This interpretation is consistent with the observed behavior and with fundamental theory. We conclude that the appropriate single scaling exponent for the size effect is $x = 1$, so that the observed strength is just the bulk strength plus a term in the inverse of the structure size in all geometries.

2. Excessive optimism in fitting

The Ramberg–Osgood empirical formula (Ramberg and Osgood, 1943) is often used to fit experimental strain-hardening data in metals. The formula, for the uniaxial tensile stress–strain curve, is often given as

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \left(\frac{\sigma}{\sigma_0}\right)^N \tag{1}$$

with $N > 1$, $\sigma_0 = E\varepsilon_0$ (E is the Youngs modulus), which is a two-parameter fit in which plasticity develops from the outset of loading – the elastic limit (yield stress σ_Y or yield strain ε_Y) is zero. It is convenient to rearrange Eq. (1) by writing $\varepsilon = \varepsilon_E + \varepsilon_P$ where $\varepsilon_E = \sigma/E$ is the elastic strain and ε_P is the plastic strain. Then Eq. (1) may be written as

$$\varepsilon_E + \varepsilon_P = \varepsilon_E + \varepsilon_0\left(\frac{\sigma}{\sigma_0}\right)^N \tag{2}$$

with $n = 1/N < 1$, from which we obtain immediately

$$\sigma = \frac{\sigma_0}{\varepsilon_0^n}\varepsilon_P^n \quad \varepsilon_E = \varepsilon_0^{1-n}\varepsilon_P^n = k\varepsilon_P^n \tag{3}$$

where the physical meaning of the work-hardening parameter k is more obvious than the meaning of σ_0 or ε_0 in Eq. (1). In fact, of course, even soft metal specimens may have a non-zero $\sigma_Y\,(\varepsilon_Y)$ due to the Peierls stress, grain size, the size effect, etc. and this is readily incorporated in Eq. (3) as

$$\varepsilon_E = \varepsilon_Y + k\varepsilon_P^n \tag{4}$$

This form is very convenient for fitting to data obtained by the load–unload method (Dunstan et al., 2011) when the plastic strain ε_P is measured directly. For comparison with stress–strain or $\varepsilon_E(\varepsilon)$ data, it would be desirable to rearrange Eq. (4) to express the elastic strain ε_E (or the stress σ) as a function of the total strain, $\varepsilon = \varepsilon_{el} + \varepsilon_{pl}$. While this can be done for particular values of n, there is no analytic solution for general n. For comparison with experiment, parametric plots of the form

$$\begin{aligned}(x,y) &= (\varepsilon,\varepsilon) && 0 < \varepsilon < \varepsilon_Y\\ (x,y) &= (\varepsilon_P + k\varepsilon_P^n, \varepsilon_Y + k\varepsilon_P^n) && \varepsilon_P > 0\end{aligned} \tag{5}$$

may be used for any n, and it is straightforward to use least-squares methods to find the best values of the parameters for the fit.

In Fig. 1 we show a typical high-quality dataset for nickel. This dataset comes in fact from foil-bending experiments, for which the stress–strain function should be integrated from the neutral plane to the maximum strain at the surface (Ehrler et al. 2008). That is not relevant or needed here: we use this data set only to show how deceptive an apparently good fit to Eq. (1) or Eq. (4) may be. In Fig. 1a the data is plotted in a form suitable for comparison with fits using Eq. (5). One of the solid lines is a fit using Eq. (5), but with $\varepsilon_Y = 0$ consistent with Eq. (1), and with a low value of the exponent, $n = 0.145$. The fit is excellent, both near the elastic line and up at the higher-strain datum. Nevertheless, the fit is far from unique. Using non-zero values of ε_Y, fits with much higher exponents of $n = 0.28$ and $n = 0.35$ fit just as well. On this plot, it is not worth looking for the difference between the three solid lines. Plotting in the form of $\varepsilon_P(\varepsilon)$ as in Fig. 1(b), we see that the significant difference between the three fits occurs only in a small range of strain around the elbow at a total strain of about 400με, and the data would have to be accurate to about 10με in plastic strain before these three fits could be experimentally distinguished. The good fit provided by Eq. (1) with $\varepsilon_Y = 0$ may mislead – it is important also to check what doesn't fit. This is very relevant to our recent paper (Dunstan et al. 2012) in which we used such fits with a fixed exponent of $x = ½$ to give values for the yield point ε_Y and the strain-hardening parameter k – one may question how much the values of these two parameters may depend on the chosen value of x.