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Effect of white layer on residual autofrettage stresses and strains in open – end cylinders

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ABSTRACT

A semi-analytic solution for residual autofrettage stresses and strains in open-end cylinders is given. The primary objective of this research is to reveal the effect of white layer in the vicinity of the inner radius of the cylinder on the magnitude of residual stresses and strains at this radius. In contrast to many available semi-analytic solutions for residual autofrettage stresses and strains, the von Mises yield criterion and its associated flow rule are adopted. A remarkable property of the solution found is that the residual stresses and strains in the white layer can be determined without solving the strain equations in the plastic region. Therefore, the solution is rather simple. In particular, numerical techniques are only necessary to solve transcendental and ordinary differential equations.

1. Introduction

“White layer” is a term referring to hard layers of material in the vicinity of surfaces. Such layers are generated during various machining and deformation processes [1]. The majority of publications devoted to white layers have been concerned with mechanisms of the generation of such layers and wear ([1–3] among many others). Influence of white layers on the development of rolling contact fatigue has been demonstrated in Ref. [4] using a numerical method and in Ref. [5] using an experimental technique. Tribological advantages of white layers have been discussed in Ref. [6]. It is of interest to understand how white layers affect structure and component performance under other loading conditions. In particular, it has been found in Ref. [2] that the elastic modulus and yield stress within white layers may increase by 170% and 390%, respectively. Therefore, it is reasonable to expect that such a huge difference in the mechanical properties between the narrow surface layer and base material affects the distribution of stresses and strains including residual stresses and strains in structures under service conditions. Analytic and semi-analytic solutions are very useful to reveal this possible effect, even though such solutions by necessity involve simplifying assumptions. One of the classical problems in the mechanics of solids is to calculate autofrettage stresses and strains in hollows cylinders. There are three main end conditions of importance, namely the closed-end, open-end and plane strain conditions. The first complete solution for the elastic perfectly plastic distribution of stresses and strains in a thick tube subject to internal pressure under plane strain conditions has been found in Ref. [7]. The Tresca yield criterion and Mises plastic flow potential have been adopted. A numerical technique is necessary to find the final solution. The same model and method of solution have been used in Ref. [8] to solve the boundary value problem for closed-end cylinders. In this work, it has been shown that the solution is a good approximation for a material yielding according to the von Mises yield criterion and its associated flow rule. Many subsequent solutions have been obtained for both hardening and non-hardening materials that obey either the von Mises or Tresca yield criteria. In particular, an exact solution for the Tresca yield criterion and its associated flow rule has been proposed in Ref. [9]. The solution is for loading and unloading of a tube subject to internal and external pressures and to temperature gradients. An arbitrary law of isotropic hardening has been included in the theory. The general solution is valid for all three main end conditions. In general a numerical technique is necessary. However, closed-form solutions are obtained when the

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strain-hardening law is linear. The range of applicability of the solution has been determined. This important requirement to exact solutions has not been included in many subsequent solutions. For example, solutions for calculating residual stresses in autofrettaged tubes have been proposed in Refs. [10,11]. The closed-form solutions for the open-end and closed-end conditions given in Ref. [10] are for perfectly plastic material behavior at loading. The plane strain solution provided in Ref. [11] is for Ludwik's power law at loading and unloading. A numerical technique is required to calculate residual stresses. The range of applicability of the solutions [10,11] has not been determined. A number of solutions for the von Mises yield criterion are available. It is often taken that the axial stress is the average of the radial and circumferential stresses [12–15]. This assumption reduces any isotropic and a large class of anisotropic pressure-independent yield criteria to the same equations from which the deviatoric stresses are found as functions of the equivalent strain or any other hardening parameter. Therefore, this final yield criterion is a particular case of the Tresca yield criterion. A similar approach in conjunction with a deformation theory of plasticity has been adopted in Ref. [16]. Numerical solutions for the von Mises yield criterion have been given in Refs. [17,18]. Loading is assumed to consist of an internal pressure in Ref. [17] and of a temperature gradient as well as an internal pressure in Ref. [18]. In both solutions unloading is purely elastic. A numerical solution for the residual autofrettage stresses based on a yield criterion depending of three stress invariants has been given in Ref. [19]. It is seen from this brief review that available solutions for the von Mises yield criterion and its associated flow rule are numerical. It is shown in the present paper that a semi-analytical solution for open-end cylinders of perfectly plastic material exists (it is impossible to derive semi-analytical solutions for the material model under consideration for the other main end conditions). Moreover, it is assumed that there is a white layer in the vicinity of the inner surface of the cylinder. The effect of this layer on the magnitude of residual stresses and strains at the inner radius of the cylinder is revealed.

2. Statement of the problem

Consider a thick-walled, elastic-plastic, open-ended cylinder with inner and outer radii s_0 and b_0, respectively subjected to internal pressure $P > 0$ and subsequent unloading. It is assumed that there is a thin hard layer in the vicinity of its inner radius. The outer radius of this layer is a_0. Therefore, its thickness is $h = a_0 - s_0$. Introduce a cylindrical coordinate system (r, θ, z) whose z-axis coincides with the axis of symmetry of the cylinder (Fig. 1). Let σ_r, σ_θ and σ_z be the normal stresses in this coordinate system. At the stage of loading the stress boundary conditions are written in the cylindrical coordinates as

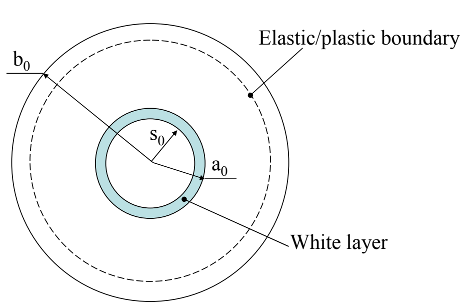

Fig. 1. Expansion of cylinder – notation.

$$\sigma_r = -P \tag{1}$$

for $r = s_0$ and

$$\sigma_r = 0 \tag{2}$$

for $r = b_0$. These boundary conditions suggest that the normal stresses in the cylindrical system of coordinates are the principal stresses. The boundary conditions at the stage of unloading will be formulated in Section 4. Strains are supposed to be small. In plastic regions, the strain tensor is assumed to be the sum of an elastic part and a plastic part. In particular, in the cylindrical coordinates

$$\varepsilon_r = \varepsilon_r^e + \varepsilon_r^p, \quad \varepsilon_\theta = \varepsilon_\theta^e + \varepsilon_\theta^p, \quad \varepsilon_z = \varepsilon_z^e + \varepsilon_z^p. \tag{3}$$

Here ε_r, ε_θ and ε_ϕ are the total normal strains, the superscript e denotes the elastic portion of the total strains and the superscript p denotes the plastic portion of the total strains. In the case of open-ended cylinders

$$\sigma_z = 0. \tag{4}$$

The elastic strains are related by Hooke's law to the stresses. Let E be Young's modulus in the range $a_0 \le r \le b_0$, E_h be Young's modulus in the range $s_0 \le r \le a_0$, and ν be Poisson's ratio in the entire cylinder. Then, under plane stress condition Hooke's law reads

$$E\varepsilon_r^e = \sigma_r - \nu\sigma_\theta, \quad E\varepsilon_\theta^e = \sigma_\theta - \nu\sigma_r, \quad E\varepsilon_z^e = -\nu(\sigma_r + \sigma_\theta) \tag{5}$$

in the range $a_0 \le r \le b_0$ and

$$E_h\varepsilon_r^e = \sigma_r - \nu\sigma_\theta, \quad E_h\varepsilon_\theta^e = \sigma_\theta - \nu\sigma_r, \quad E_h\varepsilon_z^e = -\nu(\sigma_r + \sigma_\theta) \tag{6}$$

in the range $s_0 \le r \le a_0$. The von Mises yield criterion under plane stress conditions is

$$\sigma_r^2 + \sigma_\theta^2 - \sigma_\theta\sigma_r = \sigma_0^2 \tag{7}$$

where σ_0 is the yield stress in tension in the range $a_0 \le r \le b_0$. The associated flow rule is

$$\dot{\varepsilon}_r^p = \lambda(2\sigma_r - \sigma_\theta), \quad \dot{\varepsilon}_\theta^p = \lambda(2\sigma_\theta - \sigma_r), \quad \dot{\varepsilon}_z^p = -\lambda(\sigma_\theta + \sigma_r) \tag{8}$$

where λ is a non-negative multiplier. The superimposed dot denotes the time derivative at a fixed r. The material model adopted is rate-independent. Therefore, the time derivative can be replaced with the derivative with respect to any monotonically increasing or decreasing parameter q. In particular, it is convenient to introduce the following quantities

$$\begin{gathered}
\xi_r = \frac{\partial\varepsilon_r}{\partial q}, \quad \xi_\theta = \frac{\partial\varepsilon_\theta}{\partial q}, \quad \xi_z = \frac{\partial\varepsilon_z}{\partial q}, \\
\xi_r^e = \frac{\partial\varepsilon_r^e}{\partial q}, \quad \xi_\theta^e = \frac{\partial\varepsilon_\theta^e}{\partial q}, \quad \xi_z^e = \frac{\partial\varepsilon_z^e}{\partial q}, \\
\xi_r^p = \frac{\partial\varepsilon_r^p}{\partial q}, \quad \xi_\theta^p = \frac{\partial\varepsilon_\theta^p}{\partial q}, \quad \xi_z^p = \frac{\partial\varepsilon_z^p}{\partial q}.
\end{gathered} \tag{9}$$

Then, equation (8) can be rewritten as

$$\xi_r^p = \lambda_1(2\sigma_r - \sigma_\theta), \quad \xi_\theta^p = \lambda_1(2\sigma_\theta - \sigma_r), \quad \xi_z^p = -\lambda_1(\sigma_\theta + \sigma_r) \tag{10}$$

where λ_1 is proportional to λ. The constitutive equations should be supplemented with the equilibrium equation of the form