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Comparison of the Linear Matching Method to Rolls-Royce's Hierarchical Finite Element Framework for ratchet limit analysis

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ABSTRACT

This paper provides a direct comparison between the Linear Matching Method (LMM) and the numerical procedures currently being employed within the Rolls-Royce Power Engineering (plc) Hierarchical Finite Element Framework (HFEF) for the assessment of shakedown and ratcheting behaviour. These numerical methods include the application of Direct Cyclic Analysis (DCA), utilised in an automated search procedure for load-interaction plot generation and the recently developed Hybrid procedure. The Hybrid procedure is based on a similar premise to the LMM in that the load history is decomposed into cyclic and constant components. The LMM allows for the direct evaluation of shakedown and ratchet limits to be obtained in a traditional Bree load-interaction format, along with the subsequent maximum plastic strain range for low-cycle fatigue considerations. Three problems have been used for comparison in this paper; the classic Bree cylinder, a nozzle-in-sphere with a cold media injection transient typical of nuclear power plant loading and a pressurised two-bar structure for multi-axial failure analysis. The accuracy of each method has been verified using ABAQUS step-by-step inelastic analysis. The variations in the implementation strategies associated with each method have also been discussed along with computational efficiency and effectiveness, which show that the LMM has the significant potential to improve analysis speeds via obtaining the ratchet limit boundary directly for a specified level of cyclic loading, instead of conducting an iterative search procedure.

1. Introduction

In the nuclear industry, structural integrity assessments are undertaken to the requirements of recognised international standards such as ASME III in order to provide through-life assurance against the occurrence of potential structural failure modes, including ductile burst, ratcheting and fatigue. This paper is concerned with the prediction of shakedown and prevention of the ratcheting failure mode. Ratcheting can be observed within structures operating at temperature and pressure, whereby under certain cyclic load conditions the structure accrues a net increment of plastic strain with each application of the load cycle, thus eventually leading to failure. Under certain circumstances, the load history may be such that the accumulated plastic strains cease to develop after a few initial load cycles, known as elastic shakedown.

Rolls-Royce Power Engineering plc. is currently developing the use of 'modern' finite element methods for the assessment of pressure vessel structures to the strength, shakedown and fatigue requirements of ASME III Code, Subsection NB [1]. This is encapsulated in the Hierarchical Finite Element Framework [2], or HFEF. HFEF is based on the application of Limit Load Analysis (LLA), direct shakedown prediction and nodal strain-based fatigue post-processing in lieu of the SCL and linearisation techniques traditionally used to demonstrate acceptance to the primary and secondary stress limits of ASME III Subsection NB. LLA provides assessment of the burst failure mode whilst direct shakedown prediction and strain-based fatigue analysis respectively assess the incremental collapse and fatigue failure modes. The HFEF methods do not require the application of stress classification lines (SCLs) or their associated stress classification and linearisation procedures. If the stable cyclic strain range includes a fluctuating plastic component without a net accumulation after each cycle, then reversed or alternating plasticity is observed.

Recent shakedown analysis methods have been developed by applying plasticity bounding theorems [3,4] in tandem with

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Nomenclature

$P(x,t)$	cyclic component of load history (mechanical load), at integration point x and time point t during the cycle.
$\theta(x,t)$	cyclic component of load history (thermal)
λ	load scaling factor
n	load instance number
N	the total number of time points in the cycle
$\tilde{\sigma}_{ij}$	stress tensor
$\sigma_{ij}(x,t)$	linear elastic stress history
$\tilde{\sigma}_{ij}^{\overline{F}}$	elastic stress associated with the additional constant load component
$\tilde{\sigma}_{ij}^{\Delta}(x,t)$	varying elastic stress associated with cyclic component
σ_p	mechanical load component
$\overline{\rho}_{ij}$	the constant component of the changing residual stress
$\rho_{ij}^{r}(x,t)$	the varying residual stress field
t_n	time point within the cycle
$\rho_{ij}(t_n)$	the accumulated residual stress at load instance t_n
$\Delta\varepsilon_{ij}^{n}$	the increment of plastic strain at load instance n
$\overline{\varepsilon}$	von Mises effective strain
σ_y	uniaxial yield stress of material

modern FEA packages and mathematical optimisation theory. Such methods are known as direct shakedown analysis methods and offer the advantage that the exact load history is not required for implementation purposes, only the most significant loads acting on the structure need to be specified and the shakedown theorems applied to determine a safe operational envelope in load space. The recent advances in direct shakedown approaches have led to a significant reduction in analysis times for the assessment of progressive plastic deformation in components subjected to high temperature operating conditions without sacrificing accuracy, compared to the traditional step-by-step FE methods commonly used for verification purposes. Such step-by-step methods require significant computation time for ratchet boundary prediction and cannot predict ratchet limits directly, thus only being capable of indicating if elastic shakedown, plastic shakedown or ratcheting occur for a specific load case. Direct methods can be seen to offer a supporting alternative or complete replacement of the traditional (SCL) methods, until modern computer facilities advance enough to allow step-by-step methods to become considerably more efficient. Examples of such direct methods include the Linear Matching Method [5–7], the Nonlinear Superposition Method [8], Mathematical Programming Methods [9] and Repeated Elastic Analysis methods; including Seshadri's GLOSS r-node method [10]. The LMM is distinguished from other direct methods by ensuring that equilibrium and compatibility are satisfied at each stage of analysis as well as the ability to incorporate high temperature material behaviour [6].

The LMM has recently been developed to incorporate multi-load extremes in the thermo-mechanical load domain [7] and as such offers a robust and accurate method for obtaining the ratchet boundary in a direct manner. The LMM process involves calculating the load carrying capacity of a structure subjected to a predefined cyclic load ensued by the addition of an extra constant load in order to determine the proximity of the ratchet limit. This methodology allows the stable cyclic response, i.e. the cyclic stress, residual stress and plastic strain ranges for the low-cycle fatigue assessment to be computed [7]. The entire LMM numerical procedure for both shakedown and ratcheting assessment has been incorporated into the commercial finite element code ABAQUS [11]. As a result, the LMM can be readily used for the ratchet limit assessment of structures involving load histories with complex multi-load extremes, as are commonly prevalent in the nuclear industry. A significant advantage of the LMM includes offering pressure vessel designers the ability to assess 3D structures under complex loading and allow for rapid specification of the ratchet load without requiring unrealistic computing facilities as well as ease of user implementation. Within Rolls-Royce's HFEF, two methods which are currently being developed for ratchet boundary prediction include the use of Direct Cyclic Analysis [12,13] and the Hybrid Procedure, both of which will be briefly summarised in Section 2.1.

This paper aims to compare the LMM with the application of two of Rolls-Royce's HFEF shakedown assessment methods in, Direct Cyclic Analysis (DCA) [13], as well as the recently developed Hybrid procedure [14]. The three example problems that will be used for comparison in this paper include; the classic Bree cylinder that forms the basis of the ASME III shakedown assessment [1,15], a nozzle-in-sphere with a cold-media injection transient and a pressurised two-bar structure for multi-axial failure considerations, which has been modified from Abdalla's original problem [16] via the inclusion of internal pressure. The comparison results used in this paper for the pure DCA and Hybrid methods have been obtained from Ref. [14], in order to provide benchmark data for the models analysed.

2. An overview of numerical methods for ratcheting analysis

2.1. Current numerical procedures for ratchet analysis being developed within Rolls-Royce

As mentioned, the results which have been used for comparison purposes in this paper have been derived from state of the art numerical ratchet analysis methods which are currently being employed within Rolls-Royce's HFEF. These methods are currently under development and aim to remove the uncertainties associated with the traditional ASME III shakedown assessments, which rely on traditional stress classification procedures, which can often be subjective when implemented in practical plant scenarios. Within HFEF, two methods currently which are currently being developed for ratchet boundary prediction include the use of Direct Cyclic Analysis [13] and the Hybrid procedure [14], these will be briefly explained in this section, however more in-depth details are available in Refs. [2,13,14].

DCA is a Fourier based approach which was initially implemented in ABAQUS to detect if a stabilised cyclic response existed and therefore indicate if ratcheting or elastic/plastic shakedown occurred, without determining a ratchet boundary directly [13,14]. This method however has been automated in Ref. [13] such that a ratchet boundary can be obtained via an optimisation procedure generated using Python [17], whereby varying load levels and the application of several load cycles are used to establish the ratchet boundary. The process involves using repeated DCA calculations and assessing the convergence of the solution in order to indicate if a steady cyclic response has been obtained for a particular load cycle. A bisection algorithm is used to alter the levels of loading for each DCA calculation and to search for the ratchet limit, however in order to differentiate between the strict and global shakedown limits further examination of the plastic strains must be conducted. Due to this search procedure, this method may prove to be time consuming and impractical in an industrial context for problems involving complex thermo-mechanical load histories, especially for acute thermal transients, as full details of the load cycle must be