International Journal of Pressure Vessels and Piping 107 (2013) 1–11

Contents lists available at SciVerse ScienceDirect

International Journal of Pressure Vessels and Piping

journal homepage: www.elsevier.com/locate/ijpvp

Thermal and mechanical cyclic loading of thick spherical vessels made of transversely isotropic materials

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ARTICLE INFO

Article history:
Received 30 September 2012
Received in revised form
29 December 2012
Accepted 21 January 2013

Keywords:
Kinematic hardening model
Ratcheting
Shakedown
Reversed plasticity
Stress categories

ABSTRACT

The aim of this paper is to obtain the dependency of the ratcheting, reversed plasticity, or shakedown behavior of spherical vessels made of some anisotropic materials to the stress category of imposed cyclic loading. The Hill anisotropic yield criterion with the kinematic hardening theories of plasticity based on the Prager and Armstrong–Frederick models are used to predict the yield of the vessel and obtain the plastic strains. An iterative numerical method is used to simulate the cyclic loading behavior of the structure. The effect of mean and amplitude of the mechanical and thermal loads on cyclic behavior and ratcheting rate of the vessel is investigated respectively. The ratcheting rate for the vessels made of transversely isotropic material is evaluated for the various ratios of anisotropy.

1. Introduction

Cyclic loading of structures results in structural shakedown, ratcheting, or reversed plasticity. The anisotropy of metal during the manufacturing process may affect the cyclic loading results of the structure. Extensive efforts have been made to improve the accuracy of Hill's [1,2] quadratic classical anisotropic theory of flow by many researchers such as, Hill [3–5], Caddell and Hosford [6,7], Karafillis and Bocye [8], Barlat et al. [9–12], and others. In most of these works the non-quadratic yield functions are introduced to improve the anisotropic behavior of sheet metals during the plastic works. While, the non-quadratic associated flow rules increase the complexity of the plastic analysis of the structures due to their large number of material parameters, the non-associated flow rule presented by researchers such as Stoughton [13], Staughton and Yoon [14], Aretz [15] are presented which benefits from the convenient description of anisotropic yielding and flow.

Cyclic loading effects due to imposed cyclic mechanical and thermal loads cannot be predicted by the common isotropic hardening theories, and a suitable kinematic hardening theory is needed to model the Bauschinger effect. The associated kinematic hardening proposed by Wu [16], and Hahm and Kim [17], are among the models which are capable to simulate the anisotropic properties of cyclic deformations. The kinematic hardening models proposed in these researches are based on the Prager [18] and Armstrong–Frederick [19] models.

It is possible to relate the structural behavior to the stress category in cyclic loading condition. Two stress categories that may be applied to the structure are the load and strain controlled types. According to Eslami and Shariyat [20], a load controlled stress is a normal or shear stress developed by the imposed loading which is necessary to satisfy the simple laws of equilibrium of external and internal forces and moments. It is a state of constant stress with limited final strain associated with linear elastic strain. The basic characteristic of load controlled stress is that it is not self-limiting. A deformation controlled stress is a normal or shear stress developed by the constraint of the adjacent structure. It is a state of stress with constant total strain where unlimited strain can occur causing stress reduction. The basic characteristic of the deformation controlled stress is that it is self-limiting and self-equilibrating [20]. Mahbadi and Eslami [21] investigated the cyclic loading analysis of thick vessels made of isotropic materials under load and deformation controlled conditions based on the Prager and Armstrong–Frederick kinematic hardening models. Their investigations show that the response of a structure to a cyclic loading depends on the stress category of the load applied to the structure and the hardening model used in the plastic analysis.

In this paper, cyclic loading behavior of thick spherical vessels made of some anisotropic materials under different types of loading such as thermal, mechanical, and their combinations are investigated. The materials are assumed to be anisotropic–homogeneous and obey a non-linear strain hardening law in the

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http://dx.doi.org/10.1016/j.ijpvp.2013.01.006

plastic range. The cyclic response of the structures is studied at the onset of load and deformation controlled stresses. The kinematic hardening theory based on the Prager and Armstrong–Frederick models with consideration of the Hill anisotropic yield criterion [1] and von Misses associated flow rules are used to predict the ratcheting, reverse plasticity, or shakedown behavior of the structures. The successive approximation method [22,23] is used to analyze the structural behavior under cyclic loading conditions. The results are verified with some of the available data given in the literature. The novel contribution of the present study is that, in comparison to the published works reported on cyclic loading analysis of the vessels, the effect of the material anisotropy on cyclic plasticity behavior of the structures is analyzed in this paper. Through the illustrations it is concluded that, although the cyclic loading analysis of the vessels made of isotropic materials using the Prager kinematic hardening model in both, load and deformation controlled conditions leads to reversed plasticity, using the proposed hardening model in the cyclic loading analysis of the same vessels made of some anisotropic materials, results in ratcheting.

2. Theoretical formulations

Consider a thick sphere of inside radius a and outside radius b under internal pressure P_i, and external pressure P_o. A radial temperature distribution $T(r)$ is assumed for the sphere. The dimensionless quantities are

$$\begin{aligned} &S_r = \frac{\sigma_r}{X}, S_\theta = \frac{\sigma_\theta}{X}, S_\varphi = \frac{\sigma_\varphi}{X}, e_r = \frac{\varepsilon_r}{\varepsilon_0}, e_\theta = \frac{\varepsilon_\theta}{\varepsilon_0} \\ &e_\varphi = \frac{\varepsilon_\varphi}{\varepsilon_0}, e_r^p = \frac{\varepsilon_r^p}{\varepsilon_0}, e_\theta^p = \frac{\varepsilon_\theta^p}{\varepsilon_0}, e_\varphi^p = \frac{\varepsilon_\varphi^p}{\varepsilon_0} \\ &e_r^{\text{Res}} = \frac{\varepsilon_r^{\text{Res}}}{\varepsilon_0}, e_\theta^{\text{Res}} = \frac{\varepsilon_\theta^{\text{Res}}}{\varepsilon_0}, e_\varphi^{\text{Res}} = \frac{\varepsilon_\varphi^{\text{Res}}}{\varepsilon_0} \\ &p_i = \frac{P_i}{X}, = p_0\frac{P_0}{X}, \tau_i = \frac{E_x \alpha_i T}{X}, \rho = \frac{r}{a}, \beta = \frac{b}{a} \end{aligned} \tag{1}$$

where σ_r, σ_θ and σ_φ are the radial, tangential and circumferential stresses, respectively. Similarly, ε_r, ε_θ and ε_φ are the total strains, ε_r^p, ε_θ^p and ε_φ^p are the plastic strains, and $\varepsilon_r^{\text{Res}}$, ε_θ^{Res} and $\varepsilon_\varphi^{Res}$ are the residual strains in the spherical coordinate directions, (r, θ, φ). The symbols α_i with appropriate subscripts denote the coefficients of thermal expansion associated with the corresponding directions. X is the yield stress and ε_0 is the yield strain of the thickness direction in anisotropic spheres. It is notable that in anisotropic materials the yield strengths of different directions are not the same as each other. A tensile test in the r-direction provides the yield strength X. Similarly, tensile tests in the θ- and φ-directions provide yield strengths Y and Z. Measurement of the material parameters could be performed through the compression tests; however, in anisotropic materials, the yield strengths will be the same in compression and tension [29].

It is considered that the spheres are made of orthotropic materials. Orthotropic materials are characterized by nine material coefficients, the constitutive relations for the elastic strains of such materials in the principle material axes (x, y, z) can be written as follow [24]:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{elastic} \\ \varepsilon_{yy}^{elastic} \\ \varepsilon_{zz}^{elastic} \\ \gamma_{xy}^{elastic} \\ \gamma_{xz}^{elastic} \\ \gamma_{yz}^{elastic} \end{Bmatrix} \tag{2}$$

For certain special material behavior, the anisotropic coefficients may be expressed readily in terms of engineering coefficients such as Young's module, shear module and Poisson's ratio [24]. For example, transversely isotropic materials are special kinds of orthotropic materials characterized by five material coefficients. In such materials, $(C_{22} = C_{11}, C_{23} = C_{13}, C_{66} = C_{55})$.

$(C_{11}, C_{12}, C_{13}, C_{33}, C_{55})$ are the elastic stiffness constants.

$$\begin{aligned} &C_{11} = \frac{(1 - n\nu_{zx}^2)E_x}{AB}, C_{12} = \frac{(\nu_{xy} + n\nu_{zx}^2)E_x}{AB}, C_{13} = \frac{\nu_{zx}E_x}{B}, \\ &C_{33} = \frac{(1 - \nu_{xy})E_z}{B}, C_{55} = G_{xz}, C_{44} = \frac{C_{11} - C_{12}}{2}, \end{aligned} \tag{3}$$

where

$$A = 1 + \nu_{xy} \quad , B = 1 - \nu_{xy} - 2n\nu_{zx}^2 \quad \text{and} \quad n = \frac{E_x}{E_z}$$

The ratio n is a measure of anisotropy which is known as the ratio of anisotropy. The symbols E, G and ν with appropriate subscripts denote Young's module, shear module, and Poisson's ratio associated with the corresponding axes. In this paper, it is assumed that the material is transversely isotropic with the plane of the isotropy laid in the plane of meridional and tangential directions. Accordingly, it can be concluded that in the special case of the material and structure of this study, there exists a spherical symmetry in the stress field of the medium ($\sigma_\varphi = \sigma_\theta$). It is worth mentioning that, this kind of anisotropy occurs in many cases of the structures, due to the manufacturing processes. For example, in cold-rolled structures, the elastic constants in the rolling and long-transverse directions are approximately the same [26]. Therefore, these structures may be considered as transversely isotropic materials. It is remarkable that, in this work, the subscripts (x, y, z) change into (r, θ, φ), respectively.

The equilibrium equations in the general form of spherical coordinates are:

$$\begin{aligned} &\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta r}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \sigma_{\varphi r}}{\partial_\varphi} + \frac{1}{r}\left(2\sigma_{rr} - \sigma_{\theta\theta} - \sigma_{\varphi\varphi} + \sigma_{r\theta}\cot\theta\right) = 0, \\ &\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \sigma_{\varphi\theta}}{\partial_\varphi} + \frac{1}{r}\left((\sigma_{\theta\theta} - \sigma_{\varphi\varphi}) + \cot\theta + 3\sigma_{r\theta}\right) = 0, \\ &\frac{\partial \sigma_{r\varphi}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\varphi}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \sigma_{\varphi\varphi}}{\partial_\varphi} + \frac{1}{r}\left(3\sigma_{r\varphi} + 2\sigma_{\theta\varphi}\cot\theta\right) = 0 \end{aligned} \tag{4}$$

According to spherical symmetry of the structure, stresses are only the functions of radial coordinate (r). So; the only unsatisfied equilibrium equation is:

$$\frac{\partial \sigma_r}{\partial r} = \frac{2}{r}(\sigma_\theta - \sigma_r) \tag{5}$$

and other stress components are as follow:

$$\sigma_{r\theta} = \sigma_{r\varphi} = \sigma_{\theta\varphi} = 0 \quad \text{and} \quad \sigma_\varphi = \sigma_\theta \tag{6}$$

The linear strain–displacement relations in the spherical coordinates of this work are:

$$\begin{aligned} &\varepsilon_r = \frac{\partial u}{\partial r}, \ \varepsilon_\theta = \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta}, \ \varepsilon_\varphi = \frac{1}{r\sin\theta}\frac{\partial w}{\partial \varphi} + \frac{u}{r} + \frac{v}{r}\cot\theta, \\ &\gamma_{r\theta} = \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r}, \ \gamma_{r\varphi} = \frac{1}{r\sin\theta}\frac{\partial u}{\partial \varphi} + \frac{\partial w}{\partial r} - \frac{w}{r} \end{aligned} \tag{7}$$

where u, v, w are displacement components in the spherical coordinate directions, (r, θ, φ) respectively. Considering the spherical symmetry and homogeneity conditions and making use of Eq. (2), it