International Journal of Pressure Vessels and Piping 88 (2011) 19–25

Contents lists available at ScienceDirect

International Journal of Pressure Vessels and Piping

journal homepage: www.elsevier.com/locate/ijpvp

Probabilistic safety assessment of components

E. Roos*, G. Wackenhut, R. Lammert, X. Schuler

Materials Testing Institute, University of Stuttgart, Pfaffenwaldring 32, 70569 Stuttgart, Germany

ARTICLE INFO

Article history:
Received 24 March 2010
Received in revised form
23 November 2010
Accepted 29 November 2010

Keywords:
Partial safety factors
FORM
SORM
Probabilistic fracture mechanics
R6-method
Probability of failure
Limit state function

ABSTRACT

Obviously, precautions against damage of components by operational and exceptional loads have to be taken. This is performed according to the procedures and standards of the nuclear regulations, e.g. by using the strength and fracture mechanical parameters (warranty parameters) of the involved materials. Within this integrity proof by calculation, however, the effects of the uncertainties in the input parameters cannot be evaluated quantitatively. By considering the possible parameter scatter in the geometrical dimensions and loads while performing the integrity proof, the failure probabilities, and hence the caused uncertainties, become quantifiable. In addition, postulated flaw sizes can be considered and the effects of operational measures can be evaluated. This requires, on one hand, exact knowledge about the scatter of the decisive parameters, and on the other hand knowledge about their effects on the employed methods, and hence on the result of the calculation.

In this paper the applied reliability theory is described. Furthermore, the failure probabilities, especially of the weld joints of a feed water line, were calculated and the available partial safety factors were analysed.

1. Introduction

Concerning the manufacturing and operation of safety relevant power plant components, failure must be excluded reliably. For this reason, conservative assumptions are made and adequate safety factors are chosen for the design.

Even in cases in which component design is subjected to a purely deterministic approach, a probabilistic analysis is of special value, as it allows the actual safety reserves to be estimated quantitatively.

As soon as the statistical uncertainties are identified, the failure probability or reliability of a component can be calculated. Thus, instead of using subjective evidence (empirically determined safety factors) to ensure structural integrity, one can now rely on objective evaluation criteria based on a statistical analysis of the values of interest, i.e. on partial safety factors.

Based on a flawless pipeline system, the failure probability of a component can be simplified and expressed as a function of the material state and the applied loads. A component fails if the material resistance R is smaller than the applied load. Due to the stochastic character of the parameters R and S it is necessary to qualify objectively the required safety distance between the corresponding distributions. For this purpose, the so-called partial safety factors must be determined, which can be used to guarantee proper operation.

On calculating the partial safety factors using standard procedures based on FORM/SORM, [1], one has to take into account the drawbacks of the mathematical procedure behind: determining the design point as the minimum distance in the parameter space may not find the global, but rather a local optimum. Our contribution shows, that the qualitative differences between solutions are not negligible, even if the computed quantity is nearly the same. In order to decide, which solution is the right one, additional information and knowledge about the underlying system is needed. Although the concepts from FORM/SORM are well-known, already established and the topic of many other recent publications ([2–4]), we emphasize, that the pure application of these methods may lead to unexpected results. Of course, the issue of multiple MPPs (most probable points) is known and there are efforts towards solving this problem in a general manner, but finding the correct solution still needs expert (engineering) knowledge, especially when solutions have to be evaluated and compared together.

2. Probability of failure and safety index

In our further considerations to structural safety (see e.g. [5]), it is assumed that the load S and material resistance R can be described by the stochastic independent normal distributions $N(\mu_S, \sigma_S)$ and $N(\mu_R, \sigma_R)$. Since the respective density functions are unbounded, an

* Corresponding author. Tel.: +49 (0)711/685-62604; fax: +49 (0)711/685-63144.
E-mail address: Eberhard.Roos@mpa.uni-stuttgart.de (E. Roos).

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doi:10.1016/j.ijpvp.2010.11.005

Nomenclature

α_R, α_S	weighting factors for σ_R and σ_S w.r.t. σ_G, resp.
β	safety index
γ_R, γ_S	partial safety factor of the applied load and resistance, resp.
μ_R, μ_S, μ_G	mean/expected value of R, S, G resp.
$\Phi(x)$	cumulative density function (cdf) of the standard normal distribution
σ_R, σ_S, σ_G	standard deviation of R, S, G, resp.
σ_u	ultimate tensile strength (MPa)
σ_y	yield stress; the average of the 0.2% offset yield strength σ_Y, and the ultimate tensile strength (MPa)
a	crack depth (mm)
c	crack half-length (mm)
D_o	pipe outher diameter (mm)
$f_G(g)$, $F_G(g)$	probability density function, cumulative distribution function of G, resp.
G	stochastic variable, $G\,R{-}S$, limit state definition
J_i	Rice Integral at crack initiation (kJm^2)
K_V	impact energy (J)
$K_{p,R}$, $K_{p,S}$	quantile factors of R, S, resp.
$L_{r,\max}$	plastic collapse limit load
M_b (Level D)	bending moment, service level D
$M_{b,\max}$	maximum bending moment, (kNm)
P	internal pressure, (MPa)
P_f, PoF	probability of failure
R, S	stochastic variable for resistance, applied load, resp.
r, s	resistance, applied load, resp.
r_d, s_d	values of resistance, load at the design point, resp.
r_Q, s_Q	Q -quantile of R, S, resp.
t	wall thickness (mm)
$t\prime$	nominal wall thickness (mm)

overlapping of the density functions cannot be avoided. Thus, there is always an area for which $R{-}S\,0$ is true, so that for this particular case an absolute operational safety cannot be guaranteed. As shown in Fig. 1, the overlapping area can be adjusted to fit the requirements by reducing the variances and by modifying the average values of loading or load capacity. Within this context, a differentiation is made between the central and the nominal safety zone.

The central safety zone (see Fig. 1) is described as the distance between the average values μ_S and μ_R.

In practice, the interest rather lies on the nominal safety zone, i.e. the distance between the quantiles s_Q and r_Q, since this value reflects the real distributions and genuinely describes the existing safety reserves. Reducing the s_Q and r_Q uncertainties can increase these safety reserves. As can be deduced from Fig. 1, the failure probability decreases in case of $\sigma_S^{(II)} < \sigma_S^{(I)}$. The stochastic variables R and S can be put into a functional relationship, which makes it possible to define the concept of a **limit state**.

$$G = R - S \tag{1}$$

The area $G < 0$ thereby describes the failure region and $G = 0$ corresponds to the limit curve (surface). Since loading S and material resistance R are independent and normally distributed (as assumed above), their difference is also normally distributed and it is:

$$\mu_G = \mu_R - \mu_S, \sigma_g = \sqrt{\sigma_R^2 + \sigma_S^2} \tag{2}$$

The central safety zone thereby corresponds to the distance between the average value μ_G and $g = 0$. This distance can be specified as a multiple of the standard deviation σ_G, the so-called safety index β (see Fig. 2). This safety index is equivalent to the inverse variation coefficient of the limit state function:

$$\beta = \frac{\mu_G}{\sigma_G} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{3}$$

The area beneath the density function for $G \leq 0$ corresponds to the failure probability for the operating load and is therefore denoted as the operating failure probability:

$$P_f = P_{G<0} = F_G(0) = \int_{-\infty}^{0} f_G(g)dg = \Phi\left(-\frac{\mu_G}{\sigma_G}\right) = \Phi(-\beta) \tag{4}$$

The reliability now corresponds to the survival probability and can be calculated as the complement of the operative failure probability:

$$P_r = 1 - P_f \tag{5}$$

The operating failure probability P_f and the safety index β are parameters, which are tied together by the standard normal distribution, being hence equally adequate for reliability considerations. Thereby it has to be considered that the failure probabilities are only valid over a specific time period. Table 1 shows the change of the safety index β and the operating failure probability P_f over a period of time. The values of β presented here are the required values needed in order to fulfill the given probability of failure for

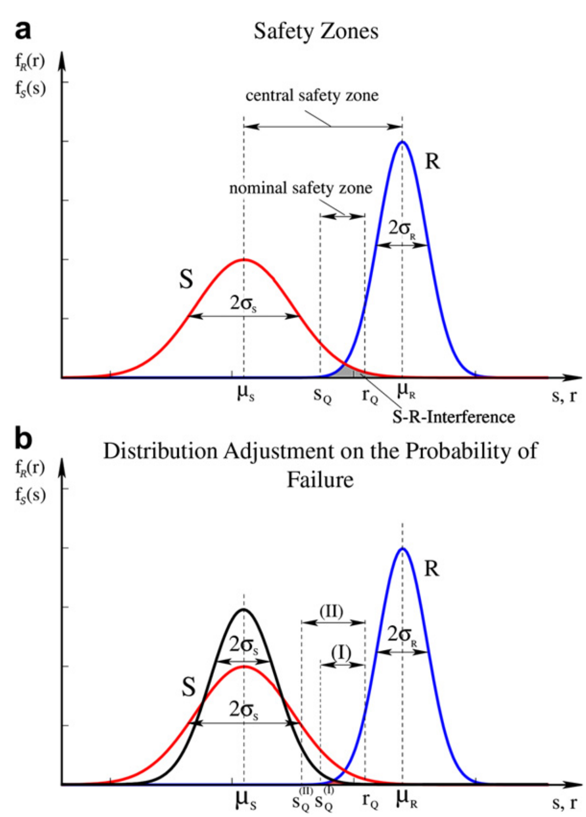

Fig. 1. Definition of the safety zones and the effect of distribution adjustment on the probability of failure, (a), Safety zones, (b), Distribution adjustment on the probability of failure.