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The effect of cooling load and thermal conductance on the local stability of an endoreversible refrigerator

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ABSTRACT

The purpose of this paper is to present a local stability analysis of an endoreversible refrigerator operating at the minimum input power P for given cooling load R absorbed from the cold reservoir, for different thermal conductances α and β, in the isothermal couplings of the working fluid with the heat reservoirs T_H and T_L ($T_H > T_L$). An endoreversible refrigerator system that is modeled by the differential equation may depend on the numerical values of certain parameters that appear in the equation. From the local stability analysis we find that a critical point of an almost linear system is a stable node. After a small perturbation the system state exponentially decays to steady state with either of two relaxation times that are a function of α, β, T_L, R and the heat capacity C. We can exhibit qualitatively the behavior of solutions of the system by sketching its phase portrait. One eigenvector in a phase portrait is the nonzero constant vector, and the other is a function of α, β, R, T_H and T_L. Finally, we discuss the local stability and energetic properties of the endoreversible refrigerator.

Impact de la charge thermique et de la conductivité thermique sur la stabilité locale d'un réfrigérateur endoréversible

Mots clés : Système frigorifique ; Système à compression ; Modélisation ; Cycle thermodynamique ; Réversibilité ; Régime permanent ; Relaxation

1. Introduction

During the last two decades, many optimization studies for refrigerators based on endoreversible and irreversible models have been performed by considering various objective functions (Grazzini, 1993; Bejan et al., 1995; Chen et al., 1995, 1998; Chen and Wu, 1996; Ait-Ali, 1996; Yan and Chen, 1996; Salah El-Din, 1999; Chen, 1999; Sahin and Kodal, 2002; Chen

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Nomenclature

b	thermal conductance ratio
C	heat capacity ($kJ\,K^{-1}$)
J	heat flow (kJ)
P	rate of power input (kW)
R	cooling load (kW)
t	cyclic time (s)
T	temperature of reservoirs (K)
W	power input (kJ)
x, y	temperatures of the working fluid (K)
z	relaxation time (s)
COP	coefficient of performance

Greek symbols

α, β	thermal conductances ($kW\,K^{-1}$)
λ	eigenvalue
μ	eigenvector

Subscripts

H	heat sink
L	heat source
max	maximum
min	minimum

Superscripts

–	overbar represent steady-state values
*	dimensionless

and Su, 2005; Ust and Sahin, 2007). Leff and Teeters (1978) have noted that the straightforward Curzon and Ahlborn's (1975) calculations will not work for a reversed Carnot cycle because there is no 'Nature maximum'. Blanchard (1980) has applied the Lagrangian method of undetermined multiplier to find out the COP of an endoreversible Carnot heat pump operated at minimum power input for a given heating load. Nevertheless, all those studies focus on the systems' steady-state properties and completely ignore their dynamic behavior. At least two different sorts of good design principles should be taken into account when building a refrigerator system: the system has to be dynamically robust, and it has to have good energetic properties (high efficiency, low entropy production, etc.). It is worthwhile to consider the effect of small perturbations or the stability of the system's steady state. This study may have good energetic steady-state properties as well as proper dynamic properties, including stability and small relaxation times.

In recent works, Santillan et al. (2001) studied the local stability of an endoreversible Curzon–Ahborn–Novikov engine operating in a maximum-power-like regime. Huang (2003) analyzed the local stability of an irreversible refrigerator working in the minimum input power for given cooling load. Guzman-Vargas et al. (2005) have shown the effect of heat-transfer laws and thermal conductances on the local stability of an endoreversible heat engine. Paez-Hernandez et al. (2006) analyzed the stability of a non-endoreversible Curzon–Ahlborn engine, taking into account the engine's implicit time delays. In this work, we analyze the stability of an endoreversible refrigerator operating in the minimum input power P for given cooling load R absorbed from the cold reservoir with Newton's linear heat-transfer law and with different thermal conductances at the isothermal branches. The influence of the numerical values of certain parameters that appear in the equation on the local stability of an endoreversible refrigerator system is discussed.

2. The steady-state properties of an endoreversible refrigerator

The model of the cycle adopted here is shown schematically in Fig. 1. The refrigerator operates in a cyclic fashion with fixed time t allotted for each cycle. The cycle of the working fluid is composed of two isothermal and two adiabatic processes. When heat transfer obeys a Newton's heat transfer law, the steady-state heat flows $\bar{J}_1$ from refrigerator to $\bar{x}$ and $\bar{J}_2$ from $\bar{y}$ to refrigerator may, respectively, be expressed as

$$\bar{J}_1 = \alpha(\bar{x} - T_H)t_1, \quad \bar{J}_2 = \beta(T_L - \bar{y})t_2 \tag{1}$$

where t_1 and t_2 are, respectively, the times of two isothermal processes at temperatures $\bar{x}$ and $\bar{y}$; α and β are, respectively, the thermal conductances between the working fluid and two heat reservoirs at temperatures T_H and T_L. Henceforth, overbars are used to indicate the corresponding variables' steady-state value. The first law says $\bar{J}_1 = \bar{J}_2 + \overline{W}$.

To obtain the simple expressions of the performance coefficient and input power, two adiabatic processes are often assumed to proceed in negligible time, such that the cycle time may be approximately given by

$$t = t_1 + t_2 \tag{2}$$

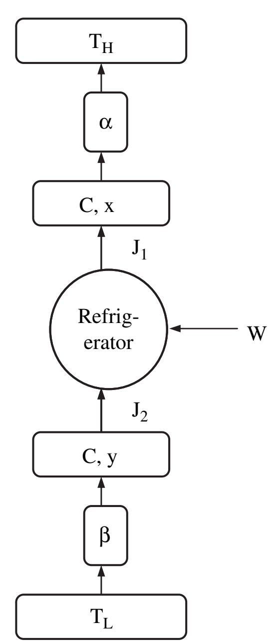

Fig. 1 – Schematic representation of an endoreversible refrigerator.