Available online at www.sciencedirect.com
ScienceDirect
International Journal of Pressure Vessels and Piping 85 (2008) 662–669
INTERNATIONAL JOURNAL OF Pressure Vessels and Piping
www.elsevier.com/locate/ijpvp

Correlated sampling techniques used in Monte Carlo simulation for risk assessment

Yun-Fu Wu*

Nuclear Safety Department, Taiwan Power Company, 242, Roosevelt Road, Section 3, Taipei, Taiwan 100, Republic of China

Received 31 January 2007; received in revised form 12 October 2007; accepted 7 November 2007

Abstract

Risk assessments in the nuclear industry heavily depend on the study of system availability/reliability and component importance. To do this study, the Monte Carlo simulation method is often a favorite selection since it involves no complex mathematical analysis, especially when systems are so complex or large that deterministic methods are difficult to solve. However, when the importance of components or the time behavior of availability/reliability of a system are required, running conventional Monte Carlo simulation alone can be very tedious and time-consuming. An integrated analysis technique that can be used to obtain the entire information efficiently and precisely in one calculation would be very desired by the system engineers. In this paper, we introduce the correlated sampling techniques to incorporate with conventional Monte Carlo simulation to save engineer's work as well as computing time.

Keywords: Risk assessment; Monte Carlo simulation; System availability/reliability; Importance; Sensitivity

1. Introduction

Most of the risk assessments in the nuclear industry involve system availability/reliability evaluation and system design optimization. Such work is often called as system reliability studies. In system reliability studies, the Monte Carlo simulation method involves no complex mathematical analysis and therefore is often a favorite selection, especially when systems are complex or large that deterministic methods have difficulty to solve. While trying to optimize a system design or to manage a system already in operation, it is often required by the system engineer to study the new system behavior many times, each time giving a small change to the system parameter such as component failure rate or repair rate, to obtain the information of component importance and sensitivity, system reliability and changed quantities. To do the work using conventional Monte Carlo simulation techniques can be very tedious and time-consuming. An integrated analysis technique that can be used to obtain the entire information efficiently and precisely in one calculation would be very desired by the system engineers. Therefore, we introduce the correlated sampling techniques to incorporate with conventional Monte Carlo simulation to fulfill the need of system engineers while doing the system reliability studies.

In this paper, firstly we brief the conventional Monte Carlo simulation method, then we introduce the scheme of correlated sampling, at last we demonstrate the function of Monte Carlo simulation incorporated with correlated sampling scheme by applying this combination to study a nuclear power plant system.

2. Monte Carlo simulation method

Monte Carlo simulation usually consists of building, with a computer program, a probabilistic model of the system under investigation. The model is then run through a large number of trials (each trial represents one history of the modeled system). From this all the information about the system performance are retrieved.

In system reliability studies, the system will be modeled as having discrete changes to the state of the components (and hence the system) arising in the continuous time.

*Tel.: +886 2 23667172; fax: +886 2 23677885.
E-mail address: u818690@taipower.com.tw

0308-0161/$ - see front matter
doi:10.1016/j.ijpvp.2007.11.004

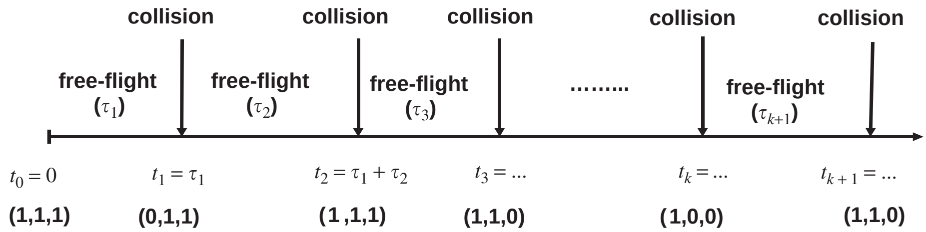

Fig. 1. Transport of a three-component system.

Events in the time development of a system are treated like neutron transport in a medium. For example, the transport behavior of a system with three components can be pictured as Fig. 1. The vector components in this figure represent the operation states ("1" represents "up" and "0" represents "down") of the components in the system. Time-segments that constitute the history of a simulated system are composed of "free-flight" periods (nothing happens in that period) and "collisions" (one of the components in the system changes its state, either fails or is repaired) at the end of the free flight.

While simulating the system transport behavior using the Monte Carlo simulation, we must pay attention to two major aspects in each time segment. One is how long does the system take to fly freely before having a collision; another is what happens at the collision. Aside from these, in system reliability studies, two other aspects need to be catered too. These are

(1) Is the new system state after collision a newly failed state or newly repaired state?
(2) Has the system passed the mission time (T_m)?

The current time of the system in simulation is the cumulative time of individual free-flight time in the past.

2.1. Free-flight time

The individual free-flight time τ_k follows the stochastic behavior of the system and is determined by generating a random number ξ_1 and is calculated as

$$\tau_k = -\ln(1-\xi_1)/\alpha_k, \tag{1}$$

where τ_k is the free flight-time before making kth collision, ξ_1 is random number uniformly distributed in [0,1), $\alpha_k = \sum_{i=1}^{n} \beta_i$, is total change rate of the system before making kth collision, here $\beta_i = \lambda_i$ (component failure rate, if component is operational); or $\beta_i = \mu_i$ (component repair rate, if component had failed); and n is the number of components in the system.

2.2. Which component changes its state at collision

The component that will make a state change at the end of a free flight is randomly chosen by generating another random number ξ_2 and comparing the following inequality equation:

$$g_{n-1} \leqslant \xi_2 < g_n, \tag{2}$$

where ξ_2 is another random number uniformly distributed in [0,1); $g_0 = 0$, and $g_n = \sum_{j=1}^{n} \beta_j/\alpha_k$, $n = 1, 2, 3, \ldots$, the sequential number of component.

As an example, if $g_2 \leqslant \xi_2 < g_3$, it can be said that component 3 has made a change at the end of the free flight.

2.3. System operability judgment

All the system reliability indices with which we are concerned are strongly relevant to the system operability. This system operability is judged by checking the developed fault-tree. A fault-tree [1] can be simply described as an analytical technique, whereby an undesirable state of the system is specified, and the system is then analyzed in the context of its environment and operation to find all credible ways in which the undesirable event can occur. The fault-tree itself is a logical model of the various parallel occurrence of the predefined undesired event and also is a complex of entities known as "gates", which serve to permit or inhibit the passage of fault logic up the tree. The gates show the relationships of events needed for the occurrence of a "higher" event. A fault-tree thus depicts the logical inter-relationship of basic events that lead to the undesired event, which is the top event of the fault-tree.

In the Monte Carlo simulation, the system operability will be checked each time that the system changes its state, i.e., a collision occurs on the way of system transport. This check-up goes through the fault-tree-logic developed for the undesired event.

2.4. Estimation of the reliability indices

To evaluate the system mean time to failure T_f, since the sum of the free-flight times at the end of history (system failed after n collisions) is the failure time of the system $(t_f)_j$ for that simulated j-history, then the average failure time after tracing N histories will be the mean time to failure of the simulated system, i.e.,

$$T_f = \frac{1}{N}\sum_{j=1}^{N}(t_f)_j. \tag{3}$$