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Interfacial stresses and fracture analysis of a two-phase composite under time-dependent heat flux

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ABSTRACT

An analysis of a two-phase composite component under time-dependent heat flux is presented. The fundamental thermoelastic solution is obtained in terms of complex potentials via the technique of the analytical continuation in order to satisfy the continuous conditions on the interface. The hereditary integral associated with the Kelvin–Maxwell model is applied to simulate the thermoviscoelastic properties while a thermorheologically simple material is considered. Based on the correspondence principle, the Laplace transformed thermoviscoelastic solution is directly determined from the corresponding thermoelastic one. The real-time solution can then be solved numerically by taking inverse Laplace transform. Some typical examples of interface stresses induced by various time-dependent heat flux are discussed. Finally, the solution of a crack embedded in the bi-material subjected to a uniform heat flux is also discussed.

1. Introduction

The stress analysis of a two-phase material under time-dependent thermal load has become an important topic because of the increasing use of composite materials in many engineering applications. Various authors [1,2] derived the appropriate forms of free energy and the corresponding stress–strain relations and dissipation energy for a thermorheologically simple material from the view point of irreversible thermodynamics. For thermoviscoelastic analysis, because of the complexity, most results reported in the literatures are found in numerical approximation [3–5]. In this paper, an analytical method to investigate the thermoviscoelastic behavior of a bonded dissimilar media subjected to thermomechanical loading is presented. The fundamental methodology of solving the thermoelastic problem is extended to solve the thermoviscoelastic problem.

In a rectangular coordinate system x_i ($i = 1, 2, 3$), let q_i, T, ε_{ij}, u_i, and σ_{ij} be the heat flux, temperature, strain, displacement, and stress, respectively. The complete set of governing equations for uncoupled thermoelastic problems involving homogeneous but anisotropic materials are [6]

$$q_i = -k_{ij}T_{,j} \tag{1}$$

$$q_{i,i} = -k_{ij}T_{,ij} = 0 \tag{2}$$

$$\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2 \tag{3}$$

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - \beta_{ij}T = c_{ijkl}u_{k,l} - \beta_{ij}T \tag{4}$$

$$\sigma_{ij,j} = c_{ijkl}u_{k,lj} - \beta_{ij}T_{,j} = 0 \tag{5}$$

where

$$k_{ij} = k_{ji}, \quad c_{ijkl} = c_{jikl} = c_{ijlk} = c_{klij}, \quad \beta_{ij} = \beta_{ji}$$

are the coefficients of heat conduction, anisotropic elastic constants and stress–temperature coefficients. The general solutions of temperature field T, total heat flux Q, displacement derivatives $\boldsymbol{u}'$ and tractions $\boldsymbol{t}$ on the x_2 plane for a homogeneous thermoelastic medium can be represented by a complex temperature function $\theta(z)$ and a complex stress function $\boldsymbol{f}(z)$ as [6–8]

$$T = \theta(z) + \bar{\theta}(\bar{z}) \tag{6}$$

$$Q = k\theta(z) + \bar{k}\bar{\theta}(\bar{z}) \tag{7}$$

$$\boldsymbol{u} = \boldsymbol{A}\boldsymbol{f}(z) + \bar{\boldsymbol{A}}\bar{\boldsymbol{f}}(\bar{z}) + \boldsymbol{d}\theta(z) + \bar{\boldsymbol{d}}\bar{\theta}(\bar{z}) \tag{8}$$

$$\boldsymbol{t} = \boldsymbol{L}\boldsymbol{f}(z) + \bar{\boldsymbol{L}}\bar{\boldsymbol{f}}(\bar{z}) + \boldsymbol{c}\theta(z) + \bar{\boldsymbol{c}}\bar{\theta}(\bar{z}) \tag{9}$$

where

$$k = k_{21} + \mu_t k_{22} \tag{10}$$

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and the anisotropic elastic constants are [7,8]

$$\boldsymbol{L}=\begin{bmatrix}-\mu_1 & -\mu_2 & -\mu_3\eta_3\\ 1 & 1 & \eta_3\\ -\eta_1 & -\eta_2 & -1\end{bmatrix},\quad \boldsymbol{A}=\begin{bmatrix}A_{11} & A_{12} & A_{13}\\ A_{21} & A_{22} & A_{23}\\ A_{31} & A_{32} & A_{33}\end{bmatrix} \tag{11}$$

with

$$\begin{aligned}
&A_{11}=s_{11}\mu_1^2+s_{12}-s_{16}\mu_1+\eta_1(s_{15}\mu_1-s_{14})\\
&A_{21}=s_{21}\mu_1+s_{22}/\mu_1-s_{26}+\eta_1(s_{25}-s_{24}/\mu_1)\\
&A_{31}=s_{41}\mu_1+s_{42}/\mu_1-s_{46}+\eta_1(s_{45}-s_{44}/\mu_1)\\
&A_{12}=s_{11}\mu_2^2+s_{12}-s_{16}\mu_2+\eta_2(s_{15}\mu_2-s_{14})\\
&A_{22}=s_{21}\mu_2+s_{22}/\mu_2-s_{26}+\eta_2(s_{25}-s_{24}/\mu_2)\\
&A_{32}=s_{41}\mu_2+s_{42}/\mu_2-s_{46}+\eta_2(s_{45}-s_{44}/\mu_2)\\
&A_{13}=\eta_3(s_{11}\mu_3^2+s_{12}-s_{16}\mu_3)+s_{15}\mu_3-s_{14}\\
&A_{23}=\eta_3(s_{21}\mu_3+s_{22}/\mu_3-s_{26})+s_{25}-s_{24}/\mu_3\\
&A_{33}=\eta_3(s_{41}\mu_3+s_{42}/\mu_3-s_{46})+s_{45}-s_{44}/\mu_3\\
&\eta_1=-l_3(\mu_1)/l_2(\mu_1),\quad \eta_2=-l_3(\mu_2)/l_2(\mu_2)\\
&\eta_3=-l_3(\mu_3)/l_4(\mu_3)
\end{aligned} \tag{12}$$

where $\varepsilon_m=s_{mn}\sigma_n+\alpha_m T$, $(m,\ n=1,\ 2,\ 6)$, μ_t is the root of the characteristic equation of conductivity with positive imaginary part, and $\mu_i (i=1,\ 2,\ 3)$ are the roots of the sixth-order characteristic equation of anisotropic compliance with positive imaginary part [6–8]. An over bar denotes the conjugate of a complex and all boldface notations indicate vector form except special statement. Moreover, $\boldsymbol{f}(z)$ can be treated as the homogeneous solution and $\theta(z)$ be the particular solution of the thermoelastic problem. Those functions will be determined exactly by means of satisfying the prescribed boundary conditions. The subscripts of z are dropped for the convenience. Once the solution is obtained for a given boundary value condition, a replacement of z_1, z_2, z_3 or z_t should be made for each component function to calculate field quantities, such as stresses, displacements or temperature. Note that the general solution is only valid when the thermal eigenvalue is different from the elastic eigenvalues. For the case that they are repeated, a small perturbation of the material constants can be employed to precede the degenerate problem.

2. Temperature field of dissimilar media

First, consider two bonded half-plane media occupying in the domain $D_b(x_2>0)$ and $D_c(x_2<0)$, respectively. The solution of singularities in a bi-material problem can be directly solved from the solution of the singularities in an infinite homogeneous medium by using the technique of analytical continuation. Suppose the singularities, designated as $\theta_0(z)$ for the same singularities embedded in an infinite homogeneous medium, are taken to be in the lower half-space of the bi-material. Then the solution of each medium can be represented as

$$\theta(z)=\begin{cases}\theta_1(z), & z\in D_b\\ \theta_0(z)+\theta_{c1}(z), & z\in D_c\end{cases} \tag{13}$$

where $\theta_1(z)$ and $\theta_{c1}(z)$ are the corresponding analytical functions in the regions D_b and D_c, which are induced by the singular function $\theta_0(z)$. Assume the bonding of the interface to be perfect, so that the temperature and the total heat flux across the interface must be continuous. It requires that

$$\begin{aligned}
\theta_1(x_1)+\bar\theta_1(x_1)&=\theta_{c1}(x_1+)\bar\theta_{c1}(x_1)+\theta_0(x_1)+\bar\theta_0(x_1)\\
k_b[\theta_1(x_1)]+\bar k_b[\bar\theta_1(x_1)]&=k_c[\theta_{c1}(x_1)+\theta_0(x_1)]\\
&\quad+\bar k_c[\bar\theta_{c1}(x_1)+\bar\theta_0(x_1)]
\end{aligned} \tag{14}$$

By the standard analytic continuation arguments it follows that

$$\begin{aligned}
\theta_1(z)&=\bar\theta_{c1}(z)+\theta_0(z),\quad z\in D_b\\
\bar\theta_1(z)&=\theta_{c1}(z)+\bar\theta_0(z),\quad z\in D_c
\end{aligned} \tag{15}$$

and

$$\begin{aligned}
k_b\theta_1(z)&=\bar k_c\bar\theta_{c1}(z)+k_c\theta_0(z),\quad z\in D_b\\
\bar k_b\bar\theta_1(z)&=k_c\theta_{c1}(z)+\bar k_c\bar\theta_0(z),\quad z\in D_c
\end{aligned} \tag{16}$$

Uncoupling Eqs. (15) and (16), we obtain

$$\begin{aligned}
\theta_1(z)&=\Pi_{bc}\theta_0(z),\quad z\in D_b\\
\theta_{c1}(z)&=\bar\Lambda_{bc}\bar\theta_0(z),\quad z\in D_c
\end{aligned} \tag{17}$$

with

$$\bar\Lambda_{bc}=\frac{\bar k_b-\bar k_c}{k_c-\bar k_b}\ \text{and}\ \Pi_{bc}=\frac{k_c-\bar k_c}{k_b-\bar k_c} \tag{18}$$

Eq. (17) give the complete temperature solution. This relation to construct a bi-material solution from a one-material solution is universal in that no specific information about the singularity is needed. Furthermore, the solution can be solved in the same method if the singularity is embedded in the upper half-space (D_b) or a singularity in a half-space interacting with a thermally insulated surface.

3. Stress field of dissimilar media

Considering an isolated singularity in the lower half-space (D_c), the stress function of the bi-material can be represented as

$$\boldsymbol{f}(z)=\begin{cases}\boldsymbol{f}_1(z), & z\in D_b\\ \boldsymbol{f}_0(z)+\boldsymbol{f}_{c1}(z), & z\in D_c\end{cases}$$

where $\boldsymbol{f}_1(z)$ and $\boldsymbol{f}_{c1}(z)$ are the corresponding analytical functions in the regions D_b and D_c, which are induced by the singular function $\boldsymbol{f}_0(z)$. Assume the bonding of the interface to be perfect, so that the displacement derivative and the traction across the interface must be continuous. It leads

$$\begin{aligned}
&\boldsymbol{L}_b\boldsymbol{f}_1(x_1)+\boldsymbol{c}_b\theta_b(x_1)++\bar{\boldsymbol{L}}_b\bar{\boldsymbol{f}}_1(x_1)+\bar{\boldsymbol{c}}_b\bar\theta_b(x_1)=\boldsymbol{L}_c\boldsymbol{f}_{c1}(x_1)\\
&+\boldsymbol{c}_c\theta_c(x_1)+\bar{\boldsymbol{L}}_c\bar{\boldsymbol{f}}_{c1}(x_1)+\bar{\boldsymbol{c}}_c\bar\theta_c(x_1)+\boldsymbol{L}_c\boldsymbol{f}_0(x_1)+\boldsymbol{c}_c\theta_0(x_1)
\end{aligned} \tag{19}$$

and

$$\begin{aligned}
&\boldsymbol{A}_b\boldsymbol{f}_1(x_1)+\boldsymbol{d}_b\theta_b(x_1)+\bar{\boldsymbol{A}}_b\bar{\boldsymbol{f}}_1(x_1)+\bar{\boldsymbol{d}}_b\bar\theta_b(x_1)=\boldsymbol{A}_c\boldsymbol{f}_{c1}(x_1)\\
&+\boldsymbol{d}_c\theta_c(x_1)+\bar{\boldsymbol{A}}_c\bar{\boldsymbol{f}}_{c1}(x_1)+\bar{\boldsymbol{d}}_c\bar\theta_c(x_1)+\boldsymbol{A}_c\boldsymbol{f}_0(x_1)+\boldsymbol{d}_c\theta_0(x_1)
\end{aligned} \tag{20}$$

Similarly, by the standard analytic continuation arguments, it becomes

$$\boldsymbol{L}_b\boldsymbol{f}_1(z)+\boldsymbol{c}_b\theta_b(z)=\bar{\boldsymbol{L}}_c\bar{\boldsymbol{f}}_{c1}(z)+\bar{\boldsymbol{c}}_c\bar\theta_c(z),\quad z\in D_b \tag{21}$$

$$\bar{\boldsymbol{L}}_b\bar{\boldsymbol{f}}_1(z)+\bar{\boldsymbol{c}}_b\bar\theta_b(z)+\boldsymbol{L}_c\boldsymbol{f}_{c1}(z)+\mathbf{c}_c\theta_c(z),\quad z\in D_c \tag{22}$$

and

$$\boldsymbol{A}_b\boldsymbol{f}_1(z)+\boldsymbol{d}_b\theta_b(z)=\bar{\boldsymbol{A}}_c\bar{\boldsymbol{f}}_{c1}(z)+\boldsymbol{d}_b\theta_b(z),\quad z\in D_b \tag{23}$$

$$\bar{\boldsymbol{A}}_b\bar{\boldsymbol{f}}_1(z)+\bar{\boldsymbol{d}}_b\bar\theta_b(z)=\boldsymbol{A}_c\boldsymbol{f}_{c1}(z)+\boldsymbol{d}_c\theta_c(z),\quad z\in D_c \tag{24}$$

Uncoupling Eqs. (21)–(24), we obtain

$$\boldsymbol{f}_1(z)=\boldsymbol{U}_{bc}\boldsymbol{f}_0(z)+\boldsymbol{f}_t(z) \tag{25}$$

$$\boldsymbol{f}_{c1}(z)=\bar{\boldsymbol{V}}_{bc}\bar{\boldsymbol{f}}_0(z)+\bar{\boldsymbol{f}}_{ct}(z) \tag{26}$$

with

$$\begin{aligned}
\boldsymbol{f}_t(z)&=\boldsymbol{\beta}_{0bc}\theta_0(z)+\boldsymbol{\beta}_{1bc}+\theta_b(z)+\bar{\boldsymbol{\gamma}}_{2bc}\bar\theta_c(z)\\
\bar{\boldsymbol{f}}_{ct}(z)&=\bar{\boldsymbol{\gamma}}_{0bc}\bar\theta_0(z)+\bar{\boldsymbol{\gamma}}_{1bc}+\bar\theta_b(z)\boldsymbol{\beta}_{2bc}\theta_c(z)
\end{aligned} \tag{27}$$