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Average shear stress yield criterion and its application to plastic collapse analysis of pipelines

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Abstract

It is known that available analytical and empirical solutions for the burst pressure of defect-free line pipes cannot broadly fit experimental data for different materials. Usually the Tresca prediction provides a lower bound to the burst pressure, and the von Mises prediction provides an upper bound to the burst pressure. A new multiaxial yield criterion, referred to as the average shear stress yield (ASSY) criterion for isotropic hardening materials, is developed in this paper based on the traditional Tresca and von Mises yield criteria so that the burst pressure of a pipeline at plastic collapse can be accurately predicted.

As an application of the proposed criterion to the plastic collapse analysis of pipelines, an ASSY-based solution for defect-free line pipes is obtained and formulated as a function of the pipe geometry, the strain hardening exponent and the ultimate tensile stress. Extensive experimental results are then adopted to validate the proposed solutions. Comparisons indicate that (1) the ASSY criterion can well fit the classical experimental data of different ductile metals for both initial and subsequent plastic yielding of a material; and (2) the ASSY-based solution of pipeline burst pressure closely matches the average experimental data for the burst pressure of defect-free pipes for various pipeline steels.

Keywords: Plastic collapse; Burst pressure; Pipeline; Material strength; Yield criterion; Tresca criterion; Mises criterion

1. Introduction

Accurate prediction of burst pressure for line pipes is crucial in the engineering design and integrity assessment of gas and oil transmission pipelines. The burst pressure is usually defined as the limit load or failure pressure of a pipe at plastic collapse, representing the maximum load-bearing capacity of the pipe. Understanding of burst pressure is very important to determine a realistic safety factor and an economic or reasonable operating pressure for pipelines. When dealing with the failure pressure prediction, the desire might be a conservative or lower bound prediction. In contrast, predictions for integrity management now focus on remaining life or defect acceptance. For such cases, prediction schemes that are conservative for pressure may produce nonconservative results for critical defect size. As small pressure errors can lead to large errors in remaining life, there is now a significant premium on accuracy. Therefore, historically accepted errors in pressure prediction are no longer acceptable, which leads to the present focus on improved consideration of multiaxial stress state effects on yield criteria so as to develop an accurate prediction of burst pressure for line pipes.

Considerable theoretical, numerical and experimental investigations have been devoted for many years to the formulation of burst pressure predictions for pipelines, and a number of analytical and empirical prediction equations were proposed for internally pressurized defect-free pipes. A comparative review of these burst pressure formulae was recently compiled by Law et al. [1] for thin-wall pipes, and by Christopher et al. [2] for thick-wall pressure vessels. They concluded that there was no one prediction method that was accurate and broadly accepted. This is not a surprise because most of them were developed for one or several specific materials. In pipeline design codes, the burst pressure of a line pipe is simply defined when the hoop stress reaches the flow stress or yield stress of the pipe

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steel. It is noted that the yield stress is used for working stress design, and the flow stress is used for plastic design. Recent experiments have indicated that the code criterion may be too conservative for modern high strength pipeline steels. Nevertheless, our previous results [3] showed that the flow stress-based prediction by code does not guarantee safety in application to modern high strength pipeline steels.

As early as in the 1950s, Cooper [4] and Svensson [5] developed a theoretical solution for predicting the burst pressure based on the von Mises yield criterion and the plastic instability theory. Hiller [6] did similar work for thin-wall tubes. However, the discussion by Clark and Woodburn attached to Cooper's paper [4] first showed that the experimental data of hoop stress for aluminium pipes at the maximum pressure were appreciably lower than the theoretical values, although they had similar trends. Kiefner et al. [7] reported a number of full-scale experimental results for extensive pipeline steels ranging from Grade B to X65 in the 1970s, including comparisons with Cooper's predictions [4]. They found that Cooper's prediction overestimates the ultimate strength for the thin-wall pipes they considered. Likewise, Rajan et al. [8] showed that the prediction of Svensson [5] overestimates their experimental data for thin-wall tubes for structural steel AISI 4130, with the discrepancy as high as 11% on average.

Cronin and Pick [9] recently examined the influence of elastic deformation on the plastic instability of line pipes. They adopted the Ramberg-Osgood material model, rather than a pure power-law material model. Using the von Mises yield criterion, these authors obtained a theoretical solution in an implicit form, and found that their prediction generally over-predicts experimental data for instability pressure of defect-free pipes. In particular, their experimental data are approximately 0.86 times their predictions for X46 pipeline steel. Similarly based on the von Mises criterion, Updike and Kalnins [10] developed a general mathematical model for limit load at tensile plastic instability for axisymmetric thin-wall pressure vessels. They showed that their calculated instability pressure was an upper bound to the burst pressure that was achieved by test. Therefore, it is evident from the analyses in Refs. [4–10] that the von Mises yield criterion may only determine an upper bound to the burst pressure for a line pipe.

The Tresca criterion is an alternative to the von Mises criterion to account for multiaxial effects on the plastic yielding and inelastic responses of a material. Based on these two yield criteria and the plastic instability theory, Steward and Klever [11] obtained two different theoretical solutions of burst pressure for defect-free pipes. They found that the experimental data of burst pressure for different ductile steels lie between predictions for the two criteria, with the von Mises prediction as an upper bound, and the Tresca prediction as a lower bound to the burst pressure. In fact, these results are consistent with those for perfectly plastic materials (Miller [12]). Furthermore, these authors found that the average of the two predictions can well match the average experimental data. These findings imply that a more rational multiaxial yield criterion for ductile materials may be needed for developing a reliable prediction of burst pressure for line pipes. As pointed out by Yu [13], more experimental results of strength of materials are obtained under complex stress states, and more accurate choices of strength theory are demanded.

Motivated by the information above, the present paper proposes a new multiaxial yield criterion for isotropic hardening materials, which is referred to as the average shear stress yield (ASSY) criterion. This new criterion is then used to predict and compare with classical experimental data for extensive metals for both initial and subsequent plastic yielding of a material. Based on the proposed criterion, a new model to predict burst pressure of defect-free pipes is obtained as a function of the pipe geometry, the strain hardening exponent and the ultimate tensile stress (UTS). To validate the proposed model, extensive experimental data of burst pressure for various pipeline steels are analysed and compared with the theoretical solutions. Based on the proposed solution, the influence of material hardening behaviour on failure pressure, the equivalent stress and the hoop stress of pipes at plastic collapse are discussed.

2. Multiaxial yield criteria

2.1. Three classical yield criteria

The Tresca criterion is the first classical yield criterion in the strength theory for isotropic ductile materials, often referred to as the maximum shear stress criterion. In principal stress space $(\sigma_1, \sigma_2, \sigma_3)$, the Tresca criterion can be expressed as

$$\tau_{\max} = \max\left(\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_2 - \sigma_3|}{2}, \frac{|\sigma_1 - \sigma_3|}{2}\right) = \frac{\sigma_0}{2}, \tag{1}$$

where $\tau_{\max}$ is the maximum shear stress and σ_0 is the yield stress in tension. For convenience, the Tresca equivalent stress, σ_T, is defined as

$$\sigma_T = \max(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|) \tag{2}$$

so that the Tresca criterion in Eq. (1) can be simply written as $\sigma_T = \sigma_0$.

The von Mises criterion is the second classical yield criterion in strength theory, often referred to as the octahedral shear stress criterion. It can be expressed by the principal stresses in the form:

$$\tau_M = \sqrt{\frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]} = \frac{\sigma_0}{\sqrt{3}}, \tag{3}$$

where τ_M is the von Mises effective shear stress. Similarly, the von Mises equivalent stress, σ_M, is defined as

$$\sigma_M = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]} \tag{4}$$