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Statistical damage theory of 2D lattices: Energetics and physical foundations of damage parameter

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Abstract

The paper presents an in-depth analysis of two-dimensional disordered lattices of statistical damage mechanics for the study of quasi-brittle materials. The strain energy variation in correspondence to damage formation is thoroughly examined and all the different contributions to the net energy changes are identified and analyzed separately. We demonstrate that the introduction of a new defect in the microstructure produces a perturbation of the microscopic random fields according to a principle of maximum energy dissipation. A redistribution parameter η is introduced to measure the load redistribution capability of the microstructure. This parameter can be estimated from simulation data of detailed models. This energetic framework sets the stage for the investigation of the statistical foundations of the damage parameter as well as the damage localization. Logical statistical arguments are developed to derive two analytical models (a maximum field model and a mean field one) for the estimate of the damage parameter via a bottom-up approach that relates the applied load to the microstructural disorder. Simulation data provided input to the statistical models as well as the means of validation. Simulated tensile tests of honeycomb lattices with mechanical disorder demonstrate that long-range interactions amongst sets of microcracks with different orientations play a fundamental role already in damage nucleation as well as in the homogeneous–heterogeneous transition. A functional "hierarchy of sets" of grain boundaries, based on their orientation in relation to the applied load, seems to emerge from this study. Results put in evidence the ability of discrete models of capturing seamlessly the damage anisotropy. The ideas exposed inhere should be useful to develop a full

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rational model for disordered lattices and, later, to extend the approach to discrete models with solid elements. The findings suggest that statistical damage mechanics might aid in the quest of reliable and physically sound constitutive relations of damage, even in synergy with micromechanics.

Keywords: Lattice models; Microstructure; Non-local effects; Damage parameter; Statistical damage

1. Introduction

Engineering materials normally experience a progressive deterioration of their mechanical and functional properties under the action of applied load and external stimuli, both during the manufacturing process and in the working environment. On the microscale this damaging process is the irreversible transformation of the microstructure via nucleation, propagation and coalescence of defects, such as microcracks and voids. Several continuum damage models have flourished in the years for ductile and brittle materials. Elasto-plastic and visco-plastic models were proposed, for example, by Simo and Ju (1987), Hansen and Schreyer (1994), Lubarda and Krajcinovic (1995), Cannmo et al. (1995), Saczuk et al. (2003), Brünig (2004), Voyiadjis et al. (2004), Bonora et al. (2005), Pierard and Doghri (2006). Other examples related to brittle solids are found in Budiansky and O'Connell (1976), Krajcinovic and Fonseka (1981a,b), Chaboche (1988a,b). Overviews of continuum damage modeling with extensive references are offered, for example, by Lemaitre (1996), Krajcinovic (1996) and Voyiadjis and Kattan (1999). The damage state is typically described by a damage parameter chosen on a phenomenological base or derived with micromechanics and homogenization techniques. Damage is intrinsically a multiscale problem where one or few micro defects in a localized region have a strong influence on the macroscale response of the solid. The presence of two length scales is an intrinsic difficulty for continuum damage modeling, more suited to capture macroscopic effects. Although "integrated" approaches exist, where coupled governing equations for microscale and macroscale are solved at once, such as the multifield theories in Mariano and Stazi (2001), the multiscale problem is often approached with a modular strategy, where separate models for the two scales are properly linked together. This approach favors the implementation of damage models within commercial finite element software. "Length scale" parameters are used in non-local continuum to transfer the microscopic changes within the microstructure to the macroscale but such phenomenological parameters are difficult to justify by a physical standpoint. The representative volume element (RVE) is typically used in micromechanics for the same purpose. However, the effective properties of an RVE depend on the averaging technique used for the global redistribution of local effects due to individual microcracks. The "dilute concentration" and self-consistent models in Krajcinovic (1996) are emblematic examples in this respect. Micromechanical models are usually adequate in the range of small damage density, as long as the damage distribution is statistical homogeneous.

The abundance of scalar, vectorial and tensorial damage parameters found in the referenced work expresses the fact that the measure of damage is not only specific to the number, shape and size of microdefects but to the underlying damage mechanism as well. Scalar damage parameters are used to model isotropic damage, e.g. Lemaitre (1985a,b), Krajcinovic and Fonseka (1981a,b) or Tvergaard (1990). The experiments by Hayhurst (1972),