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# Some remarks on the strain gradient crystal plasticity modelling, with particular reference to the material length scales involved

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**Abstract**

By exploiting the strain gradient crystal plasticity theory put forward by [Bardella, L., 2006. A deformation theory of strain gradient crystal plasticity that accounts for geometrically necessary dislocations. Journal of the Mechanics and Physics of Solids 54, 128–160], we show that a modelling involving only *energetic* material length scales through the *defect energy* (i.e., a function of Nye's dislocation density tensor added to the free energy; see, e.g., [Gurtin, M.E., 2002. A gradient theory of single-crystal viscoplasticity that accounts for geometrically necessary dislocations. Journal of the Mechanics and Physics of Solids 50, 5–32]) may not be enough in order to describe the size effect exhibited by metallic components. In fact, strain gradients that enter the constitutive modelling by taking Nye's tensor as an *independent* kinematic variable allow the description of the increase in strain hardening accompanied with diminishing size, but they do not help in capturing the related strengthening; such a size effect can be instead qualitatively described by incorporating (in a standard, phenomenological way) the gradient of the plastic slip (rate), as a further independent kinematic variable, in the isotropic hardening function that provides the resistance to flow on each slip system (Bardella, 2006). In this way, (at least) one *dissipative* length scale $L$ is introduced in the modelling, and its presence may even lead to a change in the "higher-order" (i.e., non-standard) boundary conditions to be imposed in the inherent boundary value problems. By making use of a simple example that, by taking a proper limit, also provides isotropic plasticity, we explicitly show how the nature of the relevant boundary value problems changes whether $L$ is set to zero or not, and, by analysing the modelling capability, we give an insight on the influence of the crystallography and

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conclude that it is recommendable that at least one dissipative length scale be always incorporated in the modelling.


*Keywords:* Strengthening mechanisms; Crystal plasticity; Strain gradient plasticity; Variational calculus; Grain boundaries

## 1. Introduction

We wish to investigate on the role of different length scales which can be included as material parameters into the strain gradient modelling of crystal plasticity. These material length scales are necessary for dimensional consistency when strain gradients enter the theory, and, in crystal plasticity based on a continuum description of the dislocation behaviour, are supposed to govern the processes that, in the range spanning from a few hundreds of nanometers to a few tens of micrometers, as experimentally observed,[1] lead to the size effect concerned here, with smaller being stronger.

Assuming that for monotonic loading the deformation theory context provide results close to those obtainable by means of the more appropriate flow theory counterpart,[2] we exploit the deformation theory of strain gradient crystal plasticity put forward by Bardella (2006), whose rate theory counterpart, as shown in Section 2, corresponds to an extension of the small strain modelling proposed by Gurtin and co-workers (see, e.g., Gurtin and Needleman, 2005). In Gurtin's crystal plasticity strain gradients are accounted for by an addition to the free energy called the *defect energy*, that is a function of Nye's dislocation density tensor (Nye, 1953), a kinematic variable useful to describe the peculiar size effect due to geometrically necessary dislocations (GNDs) (Ashby, 1970). This may allow one to model the variation, due to changing size, of the strain hardening and of the features of the boundary layers of plastic strain, but it seems unsuitable in order to capture the strengthening accompanied with diminishing size; this phenomenon can instead be described by adding a further strain gradient dependence, as proposed by Bardella (2006). It just consists of the analogous of the standard extension employed by many authors in the context of *isotropic plasticity*[3] in order to introduce strain gradients in the definition of the equivalent strain (rate) measure to be employed in the isotropic hardening function. In particular, in the deformation theory context, we assume that the flow resistance encountered by the glide $\gamma^{(\alpha)}$ on the slip system $\alpha$ be dependent upon the amount of "effective" plastic slip, whose definition involves the gradient of $\gamma^{(\alpha)}$

$$\gamma^{(\alpha)}_{\text{eff}} = \sqrt{\left(\gamma^{(\alpha)}\right)^2 + L^2 \gamma^{(\alpha)}_{,i} \gamma^{(\alpha)}_{,i}} \quad \forall \alpha \tag{1}$$

where $L$ is a material length scale.

[1] See, e.g., Stelmashenko et al. (1993), Fleck et al. (1994), Stölken and Evans (1998), Sun et al. (2000), and Aifantis et al. (2006).

[2] See, e.g., Budiansky (1959) and, for the gradient case, Fleck and Hutchinson (2001), Qiu et al. (2003), Fleck and Willis (2004), and Tsagrakis et al. (2006).

[3] See, e.g., Fleck and Hutchinson (2001), Gudmundson (2004), Fleck and Willis (2004), and Gurtin and Anand (2005).