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International Journal of Plasticity 22 (2006) 131–157

INTERNATIONAL JOURNAL OF
Plasticity

www.elsevier.com/locate/ijplas

An enhanced affine formulation and the corresponding numerical algorithms for the mean-field homogenization of elasto-viscoplastic composites

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Received in final revised form 3 November 2004
Available online 15 June 2005

Abstract

This paper deals with the prediction of the overall behavior of a class of two-phase elasto-viscoplastic composites, based on mean-field homogenization. For this, important improvements are made to the recently-proposed affine formulation. The latter theory linearizes the rate-dependent inelastic constitutive equations of each phase's material and transforms them into fictitious linear thermo-elastic relations in the Laplace–Carson domain. The main contributions of the present work are threefold. Firstly, complete mathematical developments including a full treatment of internal variables are carried out, enabling the modeling of the response under unloading and cyclic histories. Secondly, robust and accurate computational algorithms are proposed. Thirdly, an extensive validation of the predictions against reference unit cell finite element results is conducted for a variety of materials and loadings. A good agreement between predictions and reference results is observed.

Keywords: B. Particulate reinforced material; B. Elasto-viscoplastic material; B. Cyclic loading; C. Numerical algorithms; Homogenization

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doi:10.1016/j.ijplas.2005.04.001

1. Introduction

In order to predict the macroscopic behavior of composite materials, the formulation of mean-field homogenization models crucially depends on the mathematical structure of the constitutive equations of each phase's material. In linear elasticity, numerous homogenization schemes have been developed such as those of Voigt, Reuss, Mori-Tanaka, self-consistent and the interpolative double-inclusion model. Extensions to multi-phase composites and linear thermo-elasticity have followed (e.g., Camacho et al., 1990; Lielens, 1999; Pierard et al., 2004). In elasto-plasticity, the secant formulation gives good predictions but is limited to monotonic and proportional loadings, which is quite restrictive. In order to solve this problem, Hill (1965) linearized the local constitutive laws and introduced an instantaneous elasto-plastic tangent modulus. These relations can be homogenized at each time step according to the classical models valid in linear elasticity. However, macroscopic predictions seem to be too stiff (Gilormini, 1995). Extension of Hill's approach to rate-dependent plasticity was realized by Hutchinson (1976) who proposed a self-consistent homogenization of rigid-viscoplastic polycrystals, but again the predictions were too stiff. Similarly to Kröner (1961) in elasto-plasticity, Weng (1971) made an extension to the rate-dependent case. For this, the inelastic strain rate is viewed as a stress-free eigenstrain so that the elasto-viscoplastic material becomes similar to a linear elastic one. The main limit of this approach is that interaction between phases remains elastic, which gives again too stiff predictions. Several proposals try to avoid this problem. The transformation field analysis of Dvorak (1992) is a discretization of integral equations in which the plastic eigenstrain is piecewise uniform. An extension of this method was proposed by Chaboche et al. (2001) which uses an isotropic tangent stiffness correction in order to soften the localization rule. Sabar et al. (2002) allowed compatible fluctuations around reference values of the elastic moduli and the viscoplastic strain rate. Similarly to previous work of Paquin et al. (1999, 2001), they make use of projection operators and translated field techniques, but simultaneously instead of one after the other. Among the few attempts in finite strains are Zouhal et al. (1996) who implemented the Taylor–Lin model for polycrystals and Aboudi (2003) who has developed an incremental formulation for multiphase composites.

Molinari et al. (1987) introduced a new formulation which makes use of a tangent viscous modulus to link stress to strain rate plus an additive eigenstress. Homogenization is then performed with the linear thermo-elastic homogenization schemes. This was the first step of the so-called affine formulation which was later modified by Rougier (1994) (the modulus is no longer the tangent one and the elastic part it not neglected such as in Molinari) and more recently by Masson and Zaoui (1999) who added an implicit problem at each linearization time to avoid discontinuities of mechanical variables encountered by Rougier. The main advantages of this promising approach is that a single formulation should enable to predict accurately the macroscopic response of a wide range of non linear materials (viscoelastic, elasto-(visco)plastic, etc.). Furthermore, according to these authors, the affine formulation seems to give softer and more realistic predictions than previous proposals.