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Simplified Theory of Plastic Zones for cyclic loading and multilinear hardening

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ABSTRACT

The Simplified Theory of Plastic Zones (STPZ) is a direct method based on Zarka's method, primarily developed to estimate post-shakedown quantities of structures under cyclic loading, avoiding incremental analyses through a load histogram. In a different paper the STPZ has previously been shown to provide excellent estimates of the elastic–plastic strain ranges in the state of plastic shakedown as required for fatigue analyses. In the present paper, it is described how the STPZ can be used to predict the strains accumulated through a number of loading cycles due to a ratcheting mechanism, until either elastic or plastic shakedown is achieved, so that strain limits can be satisfied. Thus, a consistent means of estimating both, strain ranges and accumulated strains is provided for structural integrity assessment as required by pressure vessel codes. The computational costs involved typically consist of few linear elastic analyses and some local calculations. Multilinear kinematic hardening and temperature dependent yield stresses are accounted for. The quality of the results and the computational burden involved are demonstrated through four examples.

1. Introduction

Determination of elastic–plastic strain, accumulated through a number of cycles of loading due to a ratcheting mechanism, is required to satisfy strain limits set by design codes, e.g. the ASME B&PV Code [1]. Despite the rapid development of hard- and software since these strain limits were set up, it is still a challenging task. This is due to the fact that extensive human and computational resources are often required, when a step-by-step nonlinear analysis must be performed by analyzing a load histogram consisting of hundreds or thousands of loading cycles sequentially, until the state of elastic or plastic shakedown is approximately achieved. The computational burden involved may then easily add up to an equivalent of 10,000 elastic analyses. In addition to the accumulated strains, the elastic–plastic strain range is required in the case of plastic shakedown for the purpose of fatigue assessment, Fig. 1.

Some simplified methods of different complexity and accuracy are available to estimate the elastic–plastic strain range in the plastic shakedown condition without the need to perform a step-by-step elastic–plastic analyses through a load histogram. The methods most widely used in practice go back to some sort of simple knock-down factors such as the factor K_e in many nuclear design codes worldwide, or to Neuber's method [2].

Unfortunately, few simplified methods exist to predict accumulated strains in the condition of either elastic or plastic shakedown. “Direct” methods aim at predicting post-shakedown quantities without going through a load histogram on a step-by-step basis. Obviously, the path dependence of the plastic behavior of a structure gets lost. As a result, all of these methods can only provide approximations to the results obtained by cyclic incremental analysis until shakedown is achieved. In addition, the number of cycles required to reach shakedown remains unknown.

One class of direct methods is based on a sequence of linear elastic analyses by modifying Young's modulus at each location of the structure iteratively, frequently called EMAP (elastic modulus adjustment procedures). The Generalized Local Stress–Strain (GLOSS-) Method of Seshadri [3,4], the Elastic Compensation Method (ECM) of Mackenzie [5], and the Linear Matching Method (LMM) of Ponter and Chen [6,7] belong to this category. The GLOSS-method can provide an estimation of the strain range by accounting for hardening, but not of the accumulated strains. Shakedown load factors can be obtained by the ECM for a material without hardening. However, neither strain range nor accumulated strains can

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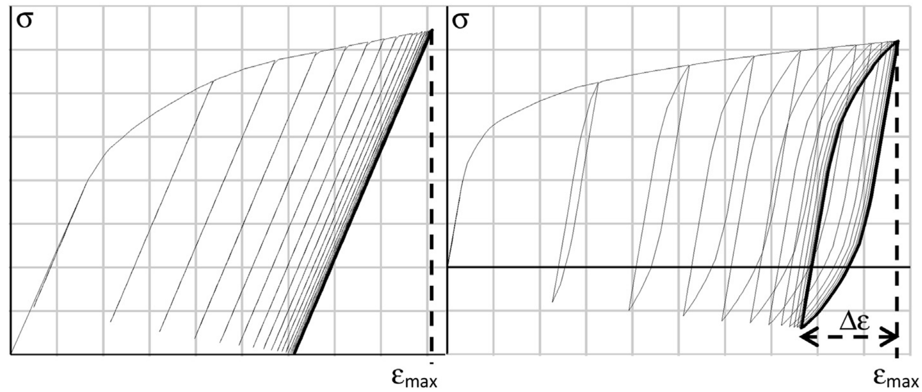

Fig. 1. Accumulated strain ε_{max} and strain range $\Delta\varepsilon$ required for life assessment; state of shakedown highlighted: elastic shakedown (left) and plastic shakedown (right).

be obtained. The LMM is able to predict strain ranges and constant strain increments per loading cycle, but is also based on the assumption of a non-hardening material.

Other direct methods, not falling into the EMAP-category, are the Large Time Increment Method (LATIN) of Ladevèze [8] and the Residual Stress Decomposition Method (RSDM) of Spiliopoulos und Panagiotou [9,10]. Similar to EMAP, Zarka's method [11,12] also makes use of a sequence of linear elastic analyses, but in a different way, namely by iteratively improving estimations of initial strains defined in the structure.

Zarka's method forms the basis of the Simplified Theory of Plastic Zones (STPZ), which was described in Ref. [13] for the purpose of obtaining strain ranges in the state of plastic shakedown. Because hardening is usually an important feature of metallic material subjected to cyclic loading, a trilinear stress–strain representation with temperature dependent yield stresses was adopted. In the present paper the STPZ is developed for predicting accumulated strains in the limit state of either elastic or plastic shakedown, so that it provides a consistent means of estimating both, strain ranges and accumulated strains. Multilinear kinematic hardening based on the Besseling model [14] is assumed. Since this model makes use of several layers of a bilinear stress–strain relationship, the STPZ for cyclic loading is presented first for linear kinematic hardening. The theoretical background of the STPZ is shown in more detail in Ref. [15].

2. STPZ for linear kinematic hardening and anisothermal cyclic loading

2.1. Nature of the shakedown state

In the case of unlimited linear kinematic hardening, Fig. 2, a structure subjected to cyclic loading will always shakedown, either

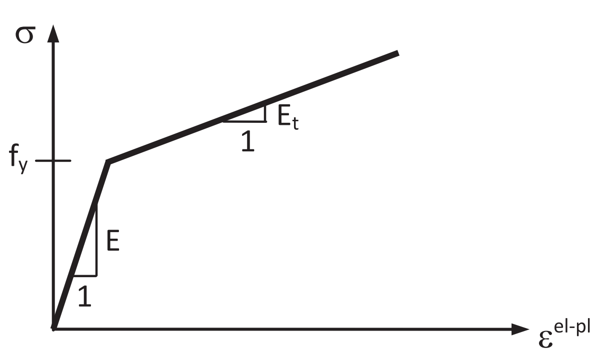

Fig. 2. Bilinear stress–strain-curve (linear kinematic hardening).

to elastic (ES) or to plastic action (PS). For a Mises yield surface and cyclic loading between two states of loading (termed minimum and maximum state of loading), the nature of shakedown can be identified on the basis of linear elastic analyses, provided strains and displacements remain small. Elastic shakedown is obtained if the Mises equivalent fictitious elastic stress range $\Delta\sigma_v^{fel}$ does not exceed twice the yield stress f_y at any location with space coordinates $\underline{x}$ in the structure of volume V. If the yield stress is different at both extremes of the loading cycle, e.g. due to temperature dependence under thermal loading, $2f_y$ may as an approximation be replaced by the sum of the corresponding yield stresses, $f_{y,min}$ and $f_{y,max}$:

$$\Delta\sigma^{fel}_{v(\underline{x})} \le \left(f_{y,min} + f_{y,max}\right)_{(\underline{x})} \quad \forall \underline{x} \in V \rightarrow ES \tag{1}$$

$$\Delta\sigma^{fel}_{v(\underline{x})} > \left(f_{y,min} + f_{y,max}\right)_{(\underline{x})} \quad \exists \underline{x} \in V \rightarrow PS. \tag{2}$$

The number of cycles required to obtain shakedown may vary between one and infinite.

2.2. Elastic shakedown

Based on Zarka's method, a transformed internal variable vector (TIV) Y_i (i = 1 … 6 for three direct and three shear components) is introduced, defined as the difference between the backstress vector ξ_i (center of yield surface in the deviatoric stress space) and the deviatoric part ρ_i' of the residual stress vector ρ_i:

$$Y_i = \xi_i - \rho_i'. \tag{3}$$

Note that all three quantities in eq. (3) vary locally but are constant with time once the structure has achieved the state of elastic shakedown. The key aspect of Zarka's method and thus also of the STPZ is that the value of the TIV can be estimated by local considerations in the state of shakedown. An approximation of the residual stress ρ_i and residual strain ε_i^* and finally the entire elastic–plastic response of the structure can then be gained by a series of linear elastic analyses with modified elastic material properties and modified loading in terms of initial stresses or strains (modified elastic analyses, MEA). For the nth MEA, the geometry of the plastic and the elastic zones, V_p and V_e respectively, is estimated by

$$V_p^{(n)} = \left\{ \underline{x} \,\middle|\, \sigma^{(n-1)}_{v,min} \ge f_{y,min} \quad \vee \quad \sigma^{(n-1)}_{v,max} \ge f_{y,max} \right\} \tag{4}$$