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Elastic properties of equilibrium foams

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ABSTRACT

Stochastic equilibrium finite element (FE) foam models are used to study the influence of relative density and distribution of solid material between cell walls and edges on the elastic properties of foam materials. It is first established that the models contain a sufficient number of cells to ascertain isotropy and numerically and statistically robust results. It is then found that the elastic moduli are very weakly coupled to cell size variation in the models, when the latter is varied systematically. The influence from relative density and distribution of solid on the elastic parameters is considerably stronger. Analytical estimates from the literature, based on idealized cell models and dimension analysis, are matched by fitting coefficients to the FE results, providing good qualitative but relatively poor quantitative correlation. An expansion of the analytical coupling functions is then suggested in order to reduce their level of idealization. The expanded formulation shows virtually perfect agreement with the numerical results for almost the whole range of relative densities and distributions of solid in the FE parameter study. The presented analytical expression is believed to be general and provide accurate estimates of the elastic properties of a wide range of foam materials, provided that their bulk material properties and micro structure can be established.

1. Introduction

A relatively simple but yet powerful model for relating the constitutive properties of solid foams to their cellular structure is the cubic model suggested by Gibson and Ashby [1]. Several scaling laws were derived from this model, relating different mechanical properties of foams to their relative density and the distribution of solid material between their cell walls and edges.

The estimates of elastic properties are based on the assumptions that the bending stiffness of cell walls can be neglected and that the cell edges only carry bending loads. Form factor coefficients are used to compensate for geometrical differences between the model and true foams. It is however generally difficult to match the models to specific real materials since there is no direct analytical way to determine the form factor coefficients for a given real foam.

Stochastic methods have been used in the past to generate realistic foam model geometries, e.g. Refs. [2–8], and to compute homogenized elastic properties taking the foam geometry, relative density and bulk material properties into account [2,4–6,8]. Stochastic models are necessary to capture the random and amorphous nature of real foam materials for which cell sizes and shapes could vary significantly. A model example from the present study is presented as two different representative volume elements (RVE) in Fig. 1, one that better illustrates the foam structure and one showing the model as implemented into the finite element (FE) software.

Such modeling approaches are however challenging since they involve a relatively high level of model complexity and require substantial and thorough computing to obtain robust and reliable results. The modeling techniques used in the work mentioned above vary some in terms of model constitutions, methods of model generation as well as in the level of mechanical sophistication when managing e.g. varying foam parameters, boundary conditions and size effects.

Stochastic foam models with different relative densities have been presented earlier but no systematic study has been found that shows how the constitutive properties scale with the relative density and the distribution of solid material in these models. The convergence of such properties with respect to the number of cells in the models has not received much attention in the past either, although a common conception seems to be that many cells are needed. In many previous studies the number of cells used in the models appear to be governed by what could be afforded computationally while the convergence as such is often not discussed.

In the present work a modeling technique specifically

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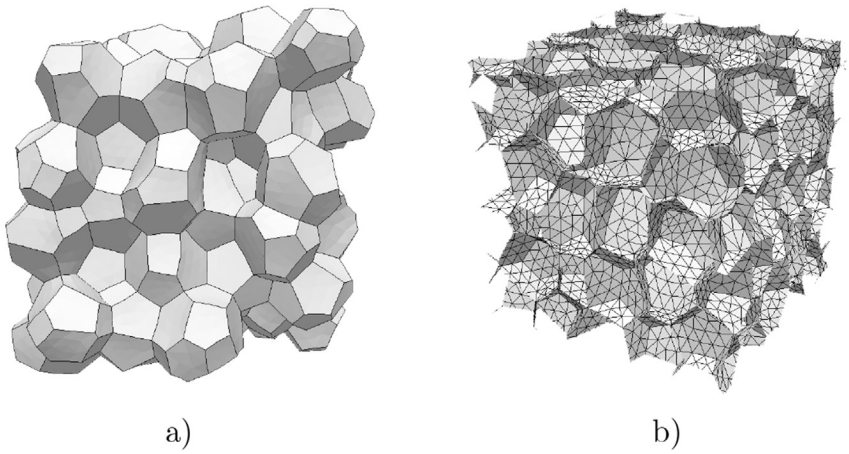

Fig. 1. Illustration of a stochastic foam model with 100 cells. a) whole-cell RVE visualizing the foam morphology and b) cubic RVE with mesh constituting the corresponding FE model.

developed to enable stochastic variation of foam parameters is used to determine constitutive properties of foam materials. Then the relations between such parameters and the homogenized mechanical properties of the models are studied. The work specifically examines the convergence of the Young's and shear moduli, and the Poisson's ratio. Then the effects from cell volume distribution, relative density and distribution of solid on these properties are investigated and presented. The results from the parameter study are used to fit coefficients to the scaling laws suggested by Gibson and Ashby [1]. A generalization of the scaling laws is finally suggested for better agreement with the modeling results over the studied ranges of relative density and distribution of solid between the cell walls and edges.

Throughout the work the models are relaxed through numerical minimization of cell surface area in combination with topology transitions, similar to the relaxation that would take place naturally in real liquid foams. An essential feature of the foam models used in the study is thus that they are at equilibrium from a surface energy point of view.

2. Method

Stochastic foam models are generated as described in a previous study by the authors [7] where the Surface Evolver software [9] is used to bring the models to surface energy equilibrium. The models are periodic in three dimensions (3D) and combinations of prescribed displacements and periodic boundary conditions are applied at all model boundaries in order to fully comply with a RVE approach.

The initial geometries come from Voronoi partitions [10] around seed points defined by the centra of equi-sized spheres at various levels of packing. The sphere populations are generated using Jodry & Tory's packing algorithm [11]. Some benefits of the algorithm are that it is computationally efficient and controlled by one single parameter only, making the procedure well defined and easily repeatable although the resulting sphere populations are random.

Surface Evolver uses a gradient method to iteratively evolve the generated Voronoi partitions into dry foams with a minimum total surface area. When the minimum is reached, topology transitions are triggered in areas where cell edges have become short and tend to vanish. Thereafter the surface area minimization is continued. These two steps are altered repeatedly until global topological convergence is reached and an equilibrium dry foam structure is obtained.

The minimization of surface area is performed without changing the volume of individual cells. The cell volume variation coming from the previous Voronoi partitioning is thus conserved.

Constitutive properties are then computed through FE analysis in Abaqus employing 6-noded Kirchoff type (STRI65) shell elements with constant thickness. The effect of material concentrations along cell edges is investigated by relocating material from the cell wall shell elements to 3-noded shear-flexible (B32) beam elements placed along the cell edges. Such struts are introduced along all cell wall edges assuming constant three-cuspid cross sections [12], illustrated in Fig. 2, where the cross section area is given by

$$A = \left(\sqrt{3} - \frac{\pi}{2}\right) r^2. \tag{1}$$

The moment of inertia of a strut is then axisymmetric and given by

$$I = \frac{1}{24}\left(20\sqrt{3} - 11\pi\right) r^4. \tag{2}$$

The thickness of the cell walls is reduced and values of A and I corresponding to the relocation of material are assigned to the beam elements along the cell edges. The cell wall material is assumed to have a Poisson's ratio $\nu_s = 0.3$ in the FE analysis. The results coming out from the models are eventually used to fit coefficients C_1, C_1', C_2 and C_2' to the scaling laws given by Gibson and Ashby [1], suggesting that

$$\frac{E^*}{E_s} = C_1 \phi^2 \left(\frac{\rho^*}{\rho_s}\right)^2 + C_1'(1-\phi)\frac{\rho^*}{\rho_s} \tag{3}$$

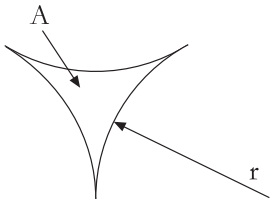

Fig. 2. Illustration of the cross-section used for the struts.