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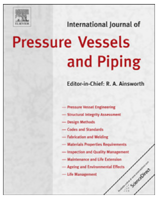

# Hygrothermoelastic responses of inhomogeneous piezoelectric and exponentially graded cylinders

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ABSTRACT

This paper presents the interaction of electric potentials, electric displacement and elastic deformations. The analytical solution is obtained to describe the hygrothermal responses in inhomogeneous piezoelectric hollow cylinders. The present cylinder is subjected to a both mechanical load and an electric potential. The displacement, stresses and electric potentials in the inhomogeneous cylinders are determined. The material properties coefficients of the present cylinder are assumed to be changed in the radial direction by different distribution forms. Two kinds of numerical application examples are displayed. The hygrothermoelastic response of a piezoelectric inhomogeneous cylinder is presented as the first kind while the hygrothermoelastic response of an exponentially graded composite cylinder is presented as the second kind. The significance of influence of different parameters is investigated. The suitable discussions and final conclusions are made.

## 1. Introduction

The stress analysis of composite structures subjected to temperature and moisture has been the subject of research interest in recent years. Composites are usually subjected to changing environmental conditions during both initial fabrication and final use. The effect of temperature is known as thermal effect while the effect of moisture absorption from the atmosphere is known as hygroscopic effect. The combined effect of temperature and moisture is known as hygrothermal effect. Heat gets conducted into the laminate when subjected to rise in the temperature. The laminate absorbs moisture when subjected to the wet conditions. The swelling or expansion is more across the fibers of the lamina. Hygrothermal effects induce a dimensional change in the laminate. But due to the mismatch of the properties of the constituents of the laminate, its free movement is inhibited. As a result, deformations and corresponding stress conditions are induced. The induced hygrothermal stresses are referred as residual stresses.

There are many works in the literature concerned with the thermoelastic analysis of various structures [1–7]. Other works are concerned with the different responses of many structures in hygrothermal environments. Studies on the effects of temperature and moisture, individually and combined, on composite materials have been carried out somewhat extensively. However, the effects of the continuous alternation of the two conditions, especially concurrently with mechanical loading, have been less explored. Whitney and Ashton [8] investigated the effects of environment on the bending of laminated composite plates. Pipes et al. [9] analyzed the stresses in laminated composite plates subjected to the combined effects of elevated temperature and absorbed moisture. Lee and Yen [10] presented the nonlinear problem of moisture and temperature effects on the stability analysis of orthotropic cylindrical shell panels subjected to axial or in-plane loading using finite element method. Lee et al. [11] presented classical laminated plate theory and von Karman's large deflection theory for laminated plate under hygrothermal conditions. The influence of hygrothermal effects on the buckling and post-buckling of shear deformable laminated plates and cylindrical shells subjected to combined loading of axial compression and external pressure are investigated by Shen [12–15]. Patel et al. [16] studied the static and dynamic characteristics of thick composite laminates exposed to hygrothermal environment using a realistic higher-order theory. Wang et al. [17] studied the response histories and distribution of interlaminar stresses in rectangular laminated plates with piezoelectric actuator layers and simply supported edges, in hygrothermal environments, subjected to free vibration and electric excitation. Zenkour [18]

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presented a hygrothermal bending analysis for a functionally graded material plate resting on elastic foundations. Lo et al. [19] developed a four-node quadrilateral plate element based on the global-local higher-order theory to study the response of laminated plates exposed to hygrothermal environment.

Piezoelectric materials have been widely used in many industrial applications due to the special electromechanical coupling effects. Many theoretical investigations have been reported in the analysis for piezoelectric composite structures with perfectly bonded interfaces. The topics include static analysis, free vibration, wave propagation and transient response. Hou et al. [20] proposed an analytical method to solve the axisymmetric plane strain electro-elasto-dynamic problem of a special non-homogeneous piezoelectric hollow cylinder subjected to dynamic loads. Dai and Wang [21] presented an analytical solution for the interaction of electric potentials, electric displacements, elastic deformations and mechanical loads, and describes electro-magneto-elastic responses and perturbation of the magnetic field vector in a piezoelectric hollow cylinder subjected to sudden mechanical load and electric potential. Wang et al. [22] obtained the transient responses in a two-layered elasto-piezoelectric composite hollow cylinder in the state of axisymmetric plane strain. Dai et al. [23] presented the analytical study for electro-magneto-thermo-elastic behaviors of a hollow cylinder composed of functionally graded piezoelectric material, placed in a uniform magnetic field, subjected to electric, thermal and mechanical loads. Wang [24] investigated the dynamic electromechanical behavior of a triple-layer piezoelectric composite cylinder with imperfect interfaces. Fesharaki et al. [25] developed the general theoretical analysis for a hollow cylinder made of functionally graded piezoelectric material subjected to two-dimensional electromechanical load.

Two problems are discussed in this paper. Both ambient temperature and moisture concentration are assumed to have variable distributions through the thickness of the cylinder. In the first problem, the thermal and moisture expansion coefficients as well as the pyroelectric coefficients are changed through the radial direction of the present cylinder. Moreover, the thermal conductivity and moisture diffusivity coefficients are also changed across the thickness of the cylinder. The hygrothermoelastic response of the piezoelectric inhomogeneous cylinder is presented.

In the second problem, a general hygrothermal stress analysis is developed in the exponentially graded composite cylinders for the axially symmetric case under general temperature and moisture distributions.

## 2. Hygrothermoelastic response of a piezoelectric cylinder

Consider a long inhomogeneous hollow cylinder of inner radius $a$ and outer radius $b$ and having perfect conductivity. Let the cylindrical coordinates of any representative point be $(r,\theta,z)$ and assume that the cylinder is subjected to a radially changing of temperature $T(r)$ and moisture concentration $C(r)$. For the axisymmetric plane strain assumption, the components of displacement, stresses, and electric displacement and electric potential may be expressed as $u(r)$, $\sigma_i(r)$, $D_r(r)$ and $\psi(r)$, respectively. The constitutive relations are:

$$\begin{Bmatrix}\sigma_r\\ \sigma_\theta\\ \sigma_z\end{Bmatrix}=\begin{bmatrix}c_{11} & c_{12} & e_{11}\\ c_{12} & c_{22} & e_{12}\\ c_{13} & c_{23} & e_{13}\end{bmatrix}\begin{Bmatrix}\dfrac{\mathrm{d}u}{\mathrm{d}r}\\ \dfrac{u}{r}\\ \dfrac{\mathrm{d}\psi}{\mathrm{d}r}\end{Bmatrix}-\begin{Bmatrix}\lambda_1\\ \lambda_2\\ 0\end{Bmatrix}T(r)-\begin{Bmatrix}\eta_1\\ \eta_2\\ 0\end{Bmatrix}C(r) \qquad (1)$$

and

$$D_r = e_{11}\frac{\mathrm{d}u}{\mathrm{d}r}+e_{12}\frac{u}{r}-\varepsilon_{11}\frac{\mathrm{d}\psi}{\mathrm{d}r}+p_{11}(r)T(r)+p_{22}(r)C(r) \qquad (2)$$

where $c_{1j}, e_{1j} (j = 1, 2, 3)$, $\varepsilon_{11}$, $p_{11}(r)$ and $p_{22}(r)$ are elastic coefficients, piezoelectric parameters, dielectric parameter and pyroelectric coefficients, respectively. In addition, $\eta_i (i = 1, 2)$ are the humidity expansion coefficients while $\lambda_i$ representing the stress-temperature moduli. They take the forms

$$\left(\begin{Bmatrix}\lambda_1\\ \lambda_2\end{Bmatrix},\begin{Bmatrix}\eta_1\\ \eta_2\end{Bmatrix}\right)=\begin{bmatrix}c_{11} & c_{12}\\ c_{12} & c_{22}\end{bmatrix}\left(\begin{Bmatrix}\alpha_1\\ \alpha_2\end{Bmatrix},\begin{Bmatrix}\beta_1\\ \beta_2\end{Bmatrix}\right) \qquad (3)$$

in which $\alpha_i$ and $\beta_i$ are the thermal and moisture expansion coefficients, respectively.

Let the thermal $\alpha_i$ and moisture $\beta_i$ expansion coefficients as well as the pyroelectric coefficients $p_{11}$ and $p_{22}$ are changed through the radial direction of the present cylinder according to the following relation:

$$P(r) = P^0\left(\frac{r}{b}\right)^{-2n}, \quad a \le r \le b \qquad (4)$$

where $n$ is a geometric parameter. Note that $P(r)$ represents the effective material properties of the inhomogeneous cylinder while $P^0$ represents the corresponding properties of the homogeneous one. The value $n$ equals zero represents a fully homogeneous cylinder. The above power-law assumption reflects a simple rule of mixtures applied only to the radial direction. The power law exponent $n$ may be varied to obtain different distributions of the components materials through the radial direction of the cylinder.

So, the radial and hoop stresses are given by

$$\begin{Bmatrix}\sigma_r\\ \sigma_\theta\end{Bmatrix}=\begin{bmatrix}c_{11} & c_{12} & e_{11}\\ c_{12} & c_{22} & e_{12}\end{bmatrix}\begin{Bmatrix}\dfrac{\mathrm{d}u}{\mathrm{d}r}\\ \dfrac{u}{r}\\ \dfrac{\mathrm{d}\psi}{\mathrm{d}r}\end{Bmatrix}-\left(\frac{r}{b}\right)^{-2n}\left(\begin{Bmatrix}\lambda_1^0\\ \lambda_2^0\end{Bmatrix}T(r)+\begin{Bmatrix}\eta_1^0\\ \eta_2^0\end{Bmatrix}C(r)\right) \qquad (5)$$

where $\lambda_i^0$ and $\eta_i^0$ are the stress-temperature moduli and humidity expansion coefficients of the homogeneous cylinder. They take the forms

$$\left(\begin{Bmatrix}\lambda_1^0\\ \lambda_2^0\end{Bmatrix},\begin{Bmatrix}\eta_1^0\\ \eta_2^0\end{Bmatrix}\right)=\begin{bmatrix}c_{11} & c_{12}\\ c_{12} & c_{22}\end{bmatrix}\left(\begin{Bmatrix}\alpha_1^0\\ \alpha_2^0\end{Bmatrix},\begin{Bmatrix}\beta_1^0\\ \beta_2^0\end{Bmatrix}\right) \qquad (6)$$

where $\alpha_i^0$ and $\beta_i^0$ are thermal expansion and moisture concentration coefficients of the homogeneous cylinder, respectively.

### 2.1. Governing equations

The temperature distribution through the radial direction of the cylinder is governed by the heat conduction equation

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\kappa\frac{\mathrm{d}}{\mathrm{d}r}T(r)\right)+q(r) = 0 \qquad (7)$$

where $\kappa(r)$ is the thermal conductivity and $q(r)$ is the heat generation function. It is to be noted that the thermal conductivity $\kappa(r)$ is also changed through the radial direction of the cylinder according to Eq. (4). The boundary conditions for temperature are

$$T(r)|_{r=a} = T_0, \quad \left.\frac{\mathrm{d}T(r)}{\mathrm{d}r}\right|_{r=b} = 0 \qquad (8)$$