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Reference load solutions for plates with semi-elliptical surface cracks subjected to biaxial tensile loading

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ABSTRACT

The yield or limit load is a key parameter with respect to the accuracy of flaw assessment based on R6 type procedures such as the R6 routine, the SINTAP and FITNET method, the standard BS 7910 and others. In a number of previous papers two of the present authors proposed the use of a reference load instead of the common limit load, which not only provided more exact fracture mechanics predictions, but showed also a wider and more general application range than the conventional parameter. Here the method has been extended to biaxial tensile loading and it has been successfully validated by a thorough comparison with finite element results and alternative solutions available in the literature.

1. Introduction

Pressurised components (e.g. pressure vessels, pipes, fuselage panels) and a number of further structures are loaded by remote biaxial tension. Some components experience even an out-of-plane loading condition resulting in a triaxial state of stress (thick-walled pipes subjected to internal pressure [1]).

Biaxiality is usually described by a biaxiality factor $\lambda = \sigma_x/\sigma_y$ and plays a crucial role in an elastic–plastic fracture mechanics analysis: i) on the material side, it has a major influence on triaxiality and therefore on the evaluation of the fracture resistance; ii) on the component side, it affects the calculation of the crack driving force by the yield or limit load in a R6 type assessment. A qualitative explanation of these effects is summarised in Table 1.

Note that the information in Table 1 is obtained from literature results [2,3]. As can be seen the effect of increasing the biaxiality is not straightforward. Qualitatively, both fracture resistance and crack driving force should be simultaneously reduced by increasing the biaxiality. However, beyond $\lambda = 0.5$ this trend reverses. As far as isotropic materials are concerned and the material follows the J_2 flow theory of plasticity, this kind of trend can be easily understood by looking at the schematic presented in Fig. 2a. The von Mises yield surface has been plotted in principal stress coordinates for the case of pure plane stress ($\sigma_{III} = 0$, $\sigma_I \neq 0$, $\sigma_{II} \neq 0$). Moreover it is important to underline that the direction *I* has been meant to be the main loading direction, therefore defining $\lambda = \sigma_{II}/\sigma_I$. Following the von Mises limit curve in Fig. 2a, for $\lambda = 0$ and $\lambda = 1$ the plastic collapse stress is equal to σ_Y, for $0 < \lambda < 1$ the biaxial state of stress increases the load carrying capacity of the structure, with a maximum set to $\lambda = 0.5$, whereas outside that range the plastic collapse stress is strongly reduced. The relationship describing the dependency of the plastic collapse stress on the biaxiality factor λ can be described as follows (in case of biaxial loading $\sigma_{III} = 0$):

$$\begin{cases} \sigma_I^2 + \sigma_{II}^2 - \sigma_I \cdot \sigma_{II} = \sigma_Y^2 \\ \sigma_{II} = \lambda \cdot \sigma_I \end{cases} \tag{1}$$

The curve described by Eq. (1) is plotted in Fig. 2b against the finite element data obtained in Ref. [2]. Note, however, that Eq. (1) holds in the pure theoretical case of flawless specimens or, in practice, in case of specimens with very small defects. This last statement can be proved as Eq. (1) approximates very well the data with $a/T = 0.2$.

In the present paper neither negative biaxiality values, nor the effect of biaxiality on fracture resistance have been taken into account, but exclusively the calculation of the crack driving force in the frame of a R6 type assessment such as R6 [4], SINTAP/FITNET [5,6], BS 7910 [7] and others, see also [8]).

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Nomenclature

a	crack depth (Fig. 1)
c	half crack length (Fig. 1)
C_i,C_{ii}	fitting parameters in the reference yield stress solutions (Eqs. (15)–(17), (21)–(25))
CDF	crack driving force
CTOD	crack tip opening displacement
E	modulus of elasticity
E'	effective modulus of elasticity (E for plane stress $E/(1-\nu^2)$; for plane strain)
$f(L_r)$	ligament yielding correction function (Eq. (14))
F	load (general notation)
F_0	reference load (Fig. 3a)
F_Y	yield or limit load (Eqs. (7) and (9))
FAD	failure assessment diagram
K	stress intensity factor
K_{mat}	fracture resistance of the material (general notation)
K_r	ordinate of the FAD diagram (Eq. (5))
K^p	plasticity-corrected K-factor (Eq. (4))
J	J-integral (Eq. (2))
J_e	elastic J-integral (Eq. (3))
L	half length of the plate (Fig. 1)
L_r	ligament yielding parameter (Eqs. (6) and (18))
R_a	ratio σ_{ref}/σ_Y as defined in Ref. [2] (Eqs. (11) and (12))
$R_{p0.2}$	0.2% proof stress
T	wall thickness of the plate (Fig. 1)
W	half width of the plate (Fig. 1)
ε	strain (general notation)
ε_{ref}	reference strain (Eq. (14))
λ	biaxiality factor $\lambda = \sigma_x/\sigma_y$
ν	Poisson's ratio
σ	stress (general notation)
σ_{app}	applied stress (Eq. (18))
σ_b	reference bending yield stress under combined tension and bending (Eq. (17))
$\sigma_{b,0}$	reference bending yield stress under pure bending (Eq. (16))
$\sigma_{biax,0}$	reference yield stress under biaxial tensile loading (Eq. (21))
σ_m	reference tensile yield stress under combined tension and bending (Eq. (17))
$\sigma_{m,0}$	reference tensile yield stress under pure tension (Eq. (15))
σ_{ref}	reference stress (Eqs. (6) and (14))
σ_0	reference yield stress (general notation, Fig. 3b)
σ_x	remote stress in x direction (Fig. 1)
σ_y	remote stress in y direction (Fig. 1)
σ_Y	yield strength (general notation), $R_{p0.2}$ in this paper
σ_i	with i = I, II, III, principal stresses

After a brief survey about the yield or limit load solutions available in the literature for semi-elliptical surface cracks subjected to biaxial loading, the concept of the reference yield stress/load will be introduced in the following. Solutions for uniaxial tensile loading, pure bending loading and combined tensile-bending loading are presented. Thereafter solutions for biaxial tensile loading are proposed and discussed thoroughly, followed by a comparison with finite element data.

2. Calculation of the crack driving force under biaxial tensile loading

Within the structural integrity assessment procedures [4–8] the crack driving force is obtained by (CDF option)

$$J = J_e \cdot [f(L_r)]^{-2} \tag{2}$$

$$\text{with } J_e = K^2 \big/ E' \tag{3}$$

and $E' = E$ for plane stress and $E' = E/(1-\nu^2)$ for plane strain. A similar expression exists for the crack tip opening displacement, CTOD. Alternatively the failure assessment diagram (FAD) option writes

$$K^p = K/f(L_r) \tag{4}$$

The resulting crack driving force is designated by K^p in Eq. (4) in order to underline that this is a plasticity-corrected K-factor. Note

Table 1
Effect of increasing remote biaxiality factor $\lambda = \sigma_x/\sigma_y$ (as presented in literature [2,3]) on both sides of a fracture mechanics analysis: the fracture resistance of the material and the crack driving force in the component.

(a) Fracture resistance of material		(b) Crack driving force in the component	
Stress triaxiality	⇧	Limit load	⇧ (⇩)
Constraint	⇧	Crack driving force in terms of J or CTOD	⇩ (⇧)
Fracture resistance	⇩