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Transient thermal stresses in functionally graded thick truncated cones by graded finite element method

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ABSTRACT

In this paper, an axisymmetric thick truncated cone made of functionally graded materials under transient thermal loads based on classical theory of linear thermoelasticity is considered. The cone is made of a combined ceramic–metal material, and its material is graded through the thickness direction according to a power law distribution. Graded finite element method based on Rayleigh–Ritz energy formulation and Crank–Nicolson algorithm is used to solve the problem in time and space domain. Distributions of temperature, displacements and stresses for different power law exponent and semi-vertex angle of the cone are investigated. Results denote that the distributions of radial displacement are qualitatively similar for the cones and cylinders but the stresses are not and due to increasing the semi-vertex angle, the nature of radial, axial and tangential stresses near the small or large bases of the cone changes. The proposed method is verified by an example which is extracted from published literature and it shows very good agreement.

1. Introduction

Functionally graded materials (FGMs) are advanced composite materials in which the material properties vary continuously from one surface to the other. The concept of FGMs was first introduced by a group of materials scientists in Japan [1]. FGMs are usually made of a combined ceramic–metal material to achieve the desired properties. The continuous variation of the material properties eliminates interface problems, minimizes thermal stress concentrations, and causes a more smooth stress distribution. FGMs have been used in numerous applications because of their high mechanical strength and high thermal resistant. Truncated hollow cones appear in many fields of engineering, such as civil, mechanical, and aerospace engineering. Therefore, it is important to study the transient thermal stresses in thick truncated hollow cones made of FGMs.

To date, a wide range of studies has been carried out on thermal stress analysis of FGM cylinders [2–28]. For example, a study of thermoelastic analysis of FGM cylindrical shells subjected to transient thermal shock loading is carried out by Santos et al. [2]. They developed a semi-analytical axisymmetric finite element model using the three-dimensional linear elasticity theory. Guo and Noda [3] using an analytical method investigated the thermal stresses of a thin FGM cylindrical shell subjected to a thermal shock. They used a perturbation method to solve the thermal diffusion equation. Ying and Wang [7] obtained an exact solution for two-dimensional elastodynamic analysis of finite hollow cylinder excited by non-uniform thermal shock. They employed the expansion of trigonometric series method and the separation of variables technique. Hosseini [8] studied the coupled thermoelasticity behavior of FG thick hollow cylinders with finite length under thermal shock load. The coupled thermoelastic equations have been considered based on Green–Naghdi theory. The cylinder was assumed to be made of many isotropic sub-cylinders across the thickness. The Galerkin Finite Element and Newmark Methods have been used to analyze the cylinder. Bahtui and Eslami [9] using a second-order shear deformation shell theory and Galerkin finite element formulation studied the coupled thermoelasticity problem of an FG cylindrical shell under impulsive thermal shock load. Kim and Noda [10] presented a Green's function approach based on the laminate theory for solving two-dimensional unsteady temperature field and associated thermal stresses in a hollow cylinder made of FGMs. Shariyat et al. [11–13] studied nonlinear thermoelasticity, vibration, and stress wave propagation analyses of plane strain thick-walled cylinders made of FGMs with temperature-dependent properties.

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Peng and Li [15,16] studied the thermoelastic problem of FGM hollow cylinders with material properties of arbitrary non-homogeneity by solving a Fredholm integral equation. Zhao et al. [17] investigated thermoelastic response and free vibration of FGM shells using the element-free kp-Ritz method. They expressed the displacement field in terms of a set of mesh-free kernel particle functions according to Sander's first-order shear deformation shell theory. Shao and Wang [18] investigated three-dimensional thermoelastic analysis of an FG cylindrical panel with finite length under non-uniform mechanical and steady state thermal loads. They obtained analytical solutions for the temperature and stress fields in terms of trigonometric and power series for the simply supported boundary conditions. Asgari and Akhlaghi [19,20] using graded finite element method studied the transient thermal stresses in a thick hollow cylinder with finite length made of two-dimensional functionally graded material based on classical theory of thermoelasticity. Shao [23] using a multilayered approach based on the theory of laminated composites, presented the solutions of temperature, displacements, and thermal/mechanical stresses in an FGM circular hollow cylinder with finite length. Safari et al. [27] presented an analytical method to study the dynamic behavior of thermoelastic stresses in a finite length FG thick hollow cylinder under thermal shock loading. They used Laplace transform and series solution to solve the thermoelastic Navier equations in displacement form.

Investigations into FGM truncated conical shells deal mainly with the thermal and mechanical buckling [29–32], free vibration and dynamic [33–41] and static analyses [42–45]. For example, Asemi et al. [41–43] utilized the finite element method to study the elastic behavior of FG thick truncated cones under hydrostatic pressures and dynamic mechanical loads.

As described in the literature, in the majority of cases the investigations are made about the steady state and time dependent thermal stresses of cylindrical shells and thick cylinders. To our knowledge, little attention is given to the analysis of thermoelastic behavior of thick truncated cones under transient thermal loads. Therefore, in this paper, classical theory of thermoelasticity and graded finite element method (GFEM) based on Rayleigh–Ritz energy formulation have been used to study the transient thermal stresses in thick truncated cones made of FGMs. In GFEM, temperature, displacements and material properties interpolated using the same shape functions and this gives continuous and smooth variation of stress field than using conventional FEM. Using this method, the effects of semi-vertex angle of the cone and the power law exponent on distribution of displacements, stresses and time responses have been considered. Also, for giving a better insight about similarities and differences between behavior of the cones and cylinders, results corresponding to cylinders with radiuses as that at the mid-height of the cones have been obtained and discussed.

2. Governing equations

2.1. Material gradient and geometry of thick truncated cone

Consider an axisymmetric thick truncated hollow cone as shown in Fig. 1, where h is the thickness of the cone, a and b are the radii of small base of the cone, L is the length and ϕ is the semi-vertex angle of the cone. r and z are the axis of cylindrical coordinate system.

The cone's material is graded through the x direction. The cone is made of a combined ceramic–metal material and the material composition varies continuously along its thickness (x direction) according to a power law distribution. The inner surface of the cone is pure ceramic and the outer surface is pure metal. The material distribution can be expressed as

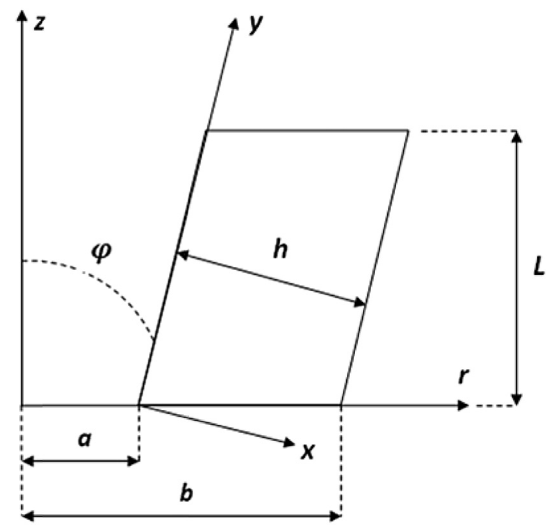

Fig. 1. Cone geometry.

$$P = P_c + (P_m - P_c)\left(\frac{x}{h}\right)^n \tag{1}$$

where P is material property such as coefficient of heat conduction k, mass density ρ, thermal expansion coefficient α, specific heat capacity C, Young's modulus E. n is a non-negative volume fraction exponent, subscripts "c" and "m" stand for ceramic and metal. x is the normal distance of points from inner surface of the cone. It should be noted the Poisson's ratio ν is assumed to be constant through the body.

x is a function of r and z directions and is evaluated using the following mathematical relations

$$x = \sqrt{(r_n - r_{in})^2 + (z_n - z_{in})^2} \tag{2}$$

$$r_{in} = \frac{\left(\tan(\pi - \varphi)r_n - z_n - a\tan\left(\frac{\pi}{2} - \varphi\right)\right)}{\tan(\pi - \varphi) - \tan\left(\frac{\pi}{2} - \varphi\right)} \tag{3}$$

$$z_{in} = \tan\left(\frac{\pi}{2} - \varphi\right)(r_{in} - a) \tag{4}$$

$$\text{For } \varphi = 0 \text{ (i.e. cylinder)} \quad x = r_n - a, h = b - a \tag{5}$$

where r_n and z_n are the radius and height of each point in the domain. r_{in} and z_{in} are the radius and height of points at the inner surface varying from a to $a + L\tan\phi$ and 0 to L, respectively.

2.2. Heat conduction equation

Consider a thick truncated hollow cone as shown in Fig. 1. The formulation reduces to two dimensions, when the boundary conditions, loads and material properties are axisymmetric, so, the formulation is independent of circumferential direction and a cylindrical coordinates (r, z) is used. Heat conduction equation in axisymmetric cylindrical coordinates when the rate of heat generation is zero is as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk_r(x)\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(k_z(x)\frac{\partial T}{\partial z}\right) = \rho(x)C(x)\partial T/\partial t \tag{6}$$

where $k_r(x)$ and $k_z(x)$ are the coefficients of heat conduction in radial and axial directions, respectively, and T denotes the temperature.