Acta Materialia 112 (2016) 295–302

Contents lists available at ScienceDirect

Acta Materialia

journal homepage: www.elsevier.com/locate/actamat

Full length article

Correlation of spherical nanoindentation stress-strain curves to simple compression stress-strain curves for elastic-plastic isotropic materials using finite element models

Dipen K. Patel, Surya R. Kalidindi*

Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332, USA

ARTICLE INFO

Article history:
Received 10 February 2016
Received in revised form
9 April 2016
Accepted 14 April 2016

Keywords:
Scaling relationship in spherical indentation
Uniaxial response
Isotropic plasticity
Finite element simulation
Constraint factor

ABSTRACT

The stress-strain fields realized in spherical indentation tests are highly heterogeneous, and present a significant challenge to the recovery of bulk stress-strain responses such as those measured in simple compression tests performed on samples with a uniform cross section in the gauge section. In this paper, we critically explore the correlations between indentation stress-strain curves and the simple compression stress-strain curves using the finite element model of indentation as a surrogate for the actual experiment. The central advantage of using a finite element model is that it allows us to critically explore the sensitivity of various assumptions or values of parameters or other choices made in the analyses protocols on the extracted results. Based on this study, a general protocol has been established to reliably recover the uniaxial stress-strain response directly from the indentation stress-strain curve for isotropic elastic-plastic materials. The protocols developed are validated for a range of hardening behaviors.

1. Introduction

Bulk mechanical properties of materials are typically measured under uniaxial loading conditions such as simple tension or simple compression [1]. However, testing methods for extracting the local properties from exceedingly small subvolumes of a sample are still in developmental stages [2–5]. Knowledge of local mechanical responses is critical for the formulation and validation of physics-based multiscale materials models [6–8]. Since the number of distinct microscale constituents (i.e., local states capturing variations in local chemistry, structure, and process history) of interest in most advanced materials is extremely large, it is highly desirable to have high throughput protocols for characterizing the local mechanical responses in hierarchical material systems.

Recent developments in instrumented indentation techniques [9–17] have resulted in the ability to probe reliably and consistently the mechanical properties of materials at the microscale. More specifically, it has been demonstrated that it is possible to extract suitably normalized indentation stress-strain (ISS) curves that display an initial linear elastic segment followed by a clear transition to a plastic response. Consequently, indentation methods have attracted considerable attention. However, relating these ISS curves to stress-strain responses measured in the conventional simple compression tests remains a significant challenge.

The central issues in the extraction of ISS curves and their correlation to simple compression stress-strain curves revolve around the definitions of the indentation stress and indentation strain measures and their correspondence with stress and strain measures used in simple compression tests. Indentation stress and strain definitions generally stem from Hertz's theory [18] for frictionless contact between two isotropic elastic solids with spherical surfaces, which may be described as

$$P = \frac{4}{3} E_{eff} R_{eff}^{\frac{1}{2}} h_e^{\frac{3}{2}}, \quad a = \sqrt{R_{eff} h_e} \tag{1}$$

where a is the contact radius at the indentation load, P, and h_e is the elastic penetration depth. R_{eff} and E_{eff} denotes the effective radius and the effective modulus, respectively, of the indenter and the specimen system. In order to convert the measured load-displacement data into ISS curve, one may define the indentation stress and indentation strain such that Hertz's theory, Eq. (1), transforms into a linear relationship as

* Corresponding author.
E-mail address: surya.kalidindi@me.gatech.edu (D.K. Patel).

http://dx.doi.org/10.1016/j.actamat.2016.04.034

$$\sigma_{ind} = \frac{4E_{eff}}{3\pi}\varepsilon_{ind}, \quad \sigma_{ind} = \frac{P}{\pi a^2}, \quad \varepsilon_{ind} = \frac{a}{R_{eff}}. \qquad (2)$$

Eq. (2) has prompted many researchers to adopt some variant of a/R_{eff} as a measure of indentation strain for the more general case of elastic-plastic indentation. In this regard, it is important to note that Eq. (2) is strictly valid only for elastic indentations. As a specific example, a majority of the studies in the literature [10,19–22] have employed the definition of indentation strain as a/R_i for elastic-plastic indentation. This definition of indentation strain as a/R_i lacks any physical interpretation as a measure of strain. This is because strain should be fundamentally interpreted as a ratio of the change in length over the initial length of a selected line segment in region of interest in the sample.

Over the past decade, various studies [23–28] have adopted the indentation stress and strain measures described above to extract ISS curves and correlate them to the uniaxial stress-strain (USS) responses. One of the earliest studies aimed at correlating spherical indentation responses to the uniaxial tests is attributed to Tabor [29]. Tabor assumes that the mean contact pressure (load divided by the projected area of the residual impression; consistent with σ_{ind} in Eq. (2)) is proportional to the "representative" flow stress of the deforming material and the corresponding "representative" strain is proportional to a/R_i (variant of the ε_{ind} in Eq. (2)). Tabor was able to empirically correlate experimentally measured values of indentation flow stresses and strains on annealed copper and mild steel with a range of spherical indenter tip radii to the stress-strain measurements in uniaxial stress conditions on the same metals. Specifically, Tabor demonstrated that the mean contact pressure (P_m) scaled by a factor of 2.8 when plotted against $0.2a/R_i$ produces an excellent match with the simple compression stress-strain curves [29]. The factor of 2.8 is defined as the "constraint" factor to capture the effect of the higher hydrostatic stress inherent to the indentation test. As mentioned earlier, and despite the remarkable agreement seen in Tabor's experiment, the definition of the indentation strain based on a/R_i holds no known physical significance. It is also important to note that in Tabor's approach, contact radius is estimated by measuring the residual impression on the unloaded sample. This definition is inconsistent with the definition used in Hertz's theory [18], which is based on the contact radius in the loaded configuration. Tabor's original approach is also very effort consuming as each indentation test produces only one data point in the plastic regime of the ISS curve.

The classical experiments of Tabor have stimulated numerous theoretical studies aimed at correlating the elastic-plastic stress-strain responses in indentation and simple compression. The theoretical treatment for the perfectly-plastic deformation imposed by a rigid frictionless indenter was explored by Ishlinsky [30] and later by Hill [31] using slip-line field approach assuming plane strain deformation (note that the real deformation mode in indentation is far from this assumption). Ishlinsky [30] performed slip-line field analysis of a spherical contact and reported that the contact pressure for a perfectly plastic contact (no hardening) is between 2.61 and 2.84. Subsequently, Hill [31] applied the slip-line field theory to the rigid plastic deformation of a flat sample with a wedge indenter, produces the widely cited result that the average pressure under the (wedge) indenter being approximately three times the flow stress in a uniaxial test. It should be noted that the imposed plane-strain boundary conditions in these theoretical approaches naturally yields an upper bound in determining the constraints factor [32].

An alternative approach to the analysis of an elastic-plastic indentation was suggested by Bishop et al. [33], and was further developed extensively by Johnson [34]. The spherical cavity model proposed by Johnson assumes that the surface of the indenter in contact is encased in a hemi-spherical core that essentially comprises both the rigid indenter and the surrounding material. The core is assumed to be in a hydrostatic stress state (i.e., the core acts as an inflating spherical cavity). Outside the core, it is assumed that the stress and displacement are radially symmetric, same as in an infinite elastic perfectly-plastic body containing a spherical cavity under pressure. The stress and displacement fields are computed invoking two conditions at the interface between the core and the elastic-plastic zone [35]: (i) the hydrostatic pressure in the core must be equal the radial component of stress in the external zone, and (ii) neglecting the compressibility in the core, the displacement of points lying on the interface during penetration must accommodate the volume of material displaced by the indenter. The spherical cavity model of indentation predicts that the mean pressure at initial yield (deviation from Hertz theory) is 1.1 times the uniaxial flow stress, and at the fully plastic state, it would reach around 3.0 times the uniaxial flow stress. Although the spherical cavity model allows imposing large plastic deformation on the sample, it neglects the upheaval or "pile-up" behavior of the material around an indenter. Also, the model assumes uniform expansion of the material around the core as in the case of spherical pressurized void in an infinite elastic-plastic space. These assumptions severely limit the utility of this approach.

In practice, it is very difficult to compute theoretically the contact stresses in an elastic-plastic indentation, because the shape and the size of the elastic-plastic boundary cannot be captured adequately in idealized simple geometries. This has led to the development of various numerical methods to the indentation simulation problem. One of the first numerical models for spherical indentation was established by Hill [25] using the infinitesimal deformation theory of plasticity. In this model, Hill invokes geometrical-similarity in the scaling of contact variables as a function of penetration depth with the spherical indenter approximated by a paraboloid of revolution. Hill's model is in excellent agreement with Tabor's findings that the representative indentation strain is $0.2a/R_i$ and that the average pressure is 2.8 times the uniaxial flow stress in tension. It should be noted that Hill's model makes the following simplifying assumptions: (i) the constitutive behavior of the indented half-space is governed by a simple power-law between suitable measures of stress and strain, and the elastic contribution to the deformation is neglected, (ii) the contact geometry remains constant throughout the indentation, and (iii) the diameter of the indenter is very large compared to the indentation depth leading to infinitesimal plastic strain imposed on the material. In spite of these simplifying assumptions, it is remarkable that the predictions of the numerical model are in excellent agreement with Tabor's experiments.

In an effort to extend the applicability of Hill's model [31] while capturing the complex heterogeneous deformation field under the indenter, several recent studies have resorted to finite element models [27,28,36,37]. A majority of these studies report a value of around 3.0 for the constraint factor for an elastic-perfectly plastic response. However, the ISS curves produced by these models show unusually large elastic-plastic transitions with high levels of apparent strain hardening (note that the materials constitutive behavior was assumed to be elastic-perfectly plastic in these models). In a recent study, Donohue et al. [38] pointed out that these abnormal features of the ISS curves arise because of the use of specific definitions of the contact radius and the indentation strain measures that are completely inconsistent with Hertz's theory. In this regard, it is important to specifically note that the modern indenters are capable of providing continuous estimates of the contact radius throughout the entire elastic-plastic loading