International Journal of Non-Linear Mechanics 85 (2016) 23–40

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Damping for large-amplitude vibrations of plates and curved panels, Part 1: Modeling and experiments

Farbod Alijani [a], Marco Amabili [b,c,*], Prabakaran Balasubramanian [b], Silvia Carra [d], Giovanni Ferrari [b], Rinaldo Garziera [c]

[a] Department of Precision of Microsystems Engineering, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands
[b] Department of Mechanical Engineering, McGill University, 817 Sherbrooke Street West, Montreal, Canada H3A 0C3
[c] Department of Industrial Engineering, University of Parma, Parco Area delle Scienze 181/A, Parma 43100, Italy
[d] Center of Excellence for Research and Innovation Technology (CERIT) – INAIL – Research, Certification and Verification Area, c/o University of Parma, Department of Industrial Engineering, Via Usberti 181/A, Parma 43124, Italy

ARTICLE INFO

Article history:
Received 9 November 2015
Received in revised form
14 May 2016
Accepted 16 May 2016
Available online 25 May 2016

Keywords:
Non-linear vibrations
Rectangular plates
Curved panels
Damping
Experiments

ABSTRACT

Theoretical and experimental non-linear vibrations of thin rectangular plates and curved panels subjected to out-of-plane harmonic excitation are investigated. Experiments have been performed on isotropic and laminated sandwich plates and panels with supported and free boundary conditions. A sophisticated measuring technique has been developed to characterize the non-linear behavior experimentally by using a Laser Doppler Vibrometer and a stepped-sine testing procedure. The theoretical approach is based on Donnell's non-linear shell theory (since the tested plates are very thin) but retaining in-plane inertia, taking into account the effect of geometric imperfections. A unified energy approach has been utilized to obtain the discretized non-linear equations of motion by using the linear natural modes of vibration. Moreover, a pseudo arc-length continuation and collocation scheme has been used to obtain the periodic solutions and perform bifurcation analysis. Comparisons between numerical simulations and the experiments show good qualitative and quantitative agreement. It is found that, in order to simulate large-amplitude vibrations, a damping value much larger than the linear modal damping should be considered. This indicates a very large and non-linear increase of damping with the increase of the excitation and vibration amplitude for plates and curved panels with different shape, boundary conditions and materials.

1. Introduction

Plates are structural elements that could be widely found in engineering applications. Flat plates with restrained displacements at the edges exhibit strong hardening type non-linearity for vibration amplitudes of the order of the plate thickness. However, the presence of imperfections in plates yield significant qualitative changes on the trend of non-linearity, and weakens the hardening response. Geometric imperfections are always present in actual plates due to manufacturing and cutting. Unlike flat plates, thin curved panels do not display hardening type non-linearity. In fact, due to the presence of initial curvature, these structures exhibit asymmetric oscillations with respect to the initial undeformed middle surface and softening non-linear behavior that turns to hardening for quite large vibration amplitudes.

The fundamental study in the analysis of large-amplitude vibrations of simply-supported rectangular plates with immovable edges was performed by Chu and Herrmann [1]. For Curved panels, Reissner [2] and Cummings [3] were pioneers in the study of large-amplitude vibrations. Since then, studying non-linear vibrations of rectangular plates and curved panels have received considerable research attention and extensive researches were performed addressing diverse non-linear phenomena in plates and panels made of traditional, composite and Functionally Graded Materials (FGMs), by using different theories and solution techniques. Here the literature on non-linear vibrations of plates and panels is not repeated but extensive reviews on this topic can be found in Sathyamoorthy [4] and Chia [5] for rectangular plates. In case of shells and panels, Amabili and Païdoussis [6] performed a detailed review until 2003 on non-linear dynamics of shells in *vacuo*, filled with or surrounded by quiescent and flowing fluids. Non-linear vibrations of shells and panels from 2003 to 2013 have been reviewed by Alijani and Amabili [7]. The monograph of

* Corresponding author at: Department of Mechanical Engineering, McGill University, 817 Sherbrooke Street West, Montreal, Canada H3A 0C3.
E-mail address: marco.amabili@mcgill.ca (M. Amabili).
URL: http://people.mcgill.ca/marco.amabili/ (M. Amabili).

http://dx.doi.org/10.1016/j.ijnonlinmec.2016.05.003
0020-7462/

Amabili [8] also provides a profound insight of the state-of-the art research on non-linear vibrations of plates, shells and curved panels. Moreover, extensive reviews on plate and shell theories made of composite materials together with governing equations and finite element formulations are provided by Carrera [9,10]. Non-linear theories of plates accounting moderate rotations and material length scales are also presented by Reddy and Srinivasa [11].

The majority of investigations on the large-amplitude vibrations of plates and panels are based on finite element method [12–16]. For instance, Reddy and Chao [12] used finite element method and first-order shear deformation theory to study large-amplitude vibrations of laminated composite plates with simply-supported and clamped boundary conditions. Ribeiro and Petyt [13,14] used the hierarchical finite element method to investigate large-amplitude vibrations of fully clamped plates without [13] and with internal resonance [14]. Ribeiro and Jansen [15] used a *p*-version finite element method to study non-linear vibrations of fully clamped cylindrical panels subjected to thermo-mechanical loading. A *p*-version finite element was also used by Akhavan and Ribeiro [16] in combination with the shooting method to study non-linear periodic vibrations of laminated plates with curved fibers. Recently, Alijani and Amabili [17,18] developed new higher-order shear deformation theories retaining the thickness deformation effect to study non-linear vibrations of pressurized plates. Particularly, in Ref. [17] several thickness deformation theories with six, seven and eight parameters were presented to study large amplitude vibrations of FGM plates by considering three-dimensional constitutive equations, and in Ref. [18] the effect of retaining full non-linear Green–Lagrange strain–displacement was investigated. In the above mentioned works either proportional damping was considered [13–16] to model the damping behavior of the structure or damping was assumed to be of the viscous type [17,18].

As it can be observed, published studies on non-linear vibrations of plates and panels are quite abundant. However, when it comes to experimental investigation, the literature is quite scarce. Non-linear vibrations of rectangular plates subjected to harmonic base excitation were investigated experimentally by Yamaki et al. [19]. Nguyen et al. [20] performed experimental tests to study the dynamic stability of rectangular plates having different boundary conditions and subjected to periodic in-plane loads. Theory and experiments for non-linear vibrations of rectangular plates with supported and free edges were presented by Amabili [21] and Alijani and Amabili [22], respectively. In these works, numerical simulations were performed by using a Lagrangian approach including viscous damping, and numerical results were obtained by using a pseudo arc-length continuation technique. Experiments and simulations were conducted by Amabili and Carra [23] to investigate the effect of concentrated masses on the non-linear vibrations of fully clamped rectangular plates. Non-linear vibrations of clamped rectangular plates in contact with sloshing liquid were investigated experimentally by Carra et al. [24]. Experiments on large-amplitude vibrations of composite and sandwich panels are due to Harras et al. [25] and Alijani et al. [26]. Particularly, non-linear free vibration response of fully clamped composite fiber reinforced and Glare 3 hybrid panels were investigated in [25], and non-linear forced vibrations of sandwich panels free at all edges were presented in [26].

Experiments on non-linear vibrations of cylindrical panels have been conducted by Amabili et al. [27], Amabili [28], and Kurpa et al. [29]. In particular, panels with zero transverse displacement and free in-plane motion at straight edges were tested in [27,28], while panels having complex base were tested in [29]. Non-linear vibration experiments on circular cylindrical shells were conducted by Amabili [30] and Pellicano [31,32].

In the present study, non-linear vibrations of plates and curved panels are revisited by performing new experimental tests and obtaining new accurate reduced-order models based on Lagrangian approach. Detailed non-linear experimental results are presented for five cases: (i) a stainless steel rectangular plate with free edges; (ii) a sandwich plate with carbon/epoxy skins and foam core; (iii) a second sandwich plate with carbon/epoxy skins and honeycomb core; (iv) a stainless steel rectangular plate with supported edges; (v) a cylindrical stainless steel panel with simply supported boundary conditions. In particular, the new set of experiments performed on isotropic and sandwich plates were meant to gain vibration amplitudes up to 1.5 times the thickness and in some cases (i.e. for the free-edge isotropic plate) nearly 4 times the thickness. To conduct non-linear tests, a laser Doppler vibrometer and a LMS signal processing and data acquisition system are used to obtain non-linear frequency–response curves. Particularly, a stepped-sine testing algorithm has been implemented by increasing and decreasing the excitation frequency in the spectral neighborhood of the lowest natural frequencies in very small steps and at specific force amplitudes controlled in a closed-loop. The theoretical reduced-order models are built via multi-modal Lagrangian approach and by using the Donnell's non-linear shell theory, since the plates and the panels are very thin, but retaining in-plane inertia (for the composite sandwich plates a shear deformation theory has also been used for comparison and it has given similar results [26]). Geometric imperfections are taken into account and are identified by measuring the deviations of the plates/panels from the ideal shape. The non-linear set of ordinary differential equations is solved by using a code based on pseudo arc-length continuation and collocation technique. Results show good qualitative and quantitative agreement between experimental and numerical non-linear frequency-response curves, verifying the accuracy of the reduced-order models. Moreover, it is revealed that in order to simulate very large amplitude vibrations and match the experimental peak amplitudes, a modal viscous damping value much larger than the one identified by linear modal analysis should be considered. This damping behavior indicates the importance of considering non-linear damping models in studying large-amplitude vibrations of plates and panels in future studies. The dependence of the damping ratio from the vibration amplitude is identified and compared for the discussed experiments in Part 2 of the present study.

2. Basic equations

Fig. 1 shows a curved panel of rectangular base with length a, uniform thickness h and constant mean radius R in curvilinear coordinate system (x, $y = R\theta$, z). According to Donnell's shallow shell theory [8], the strain components ε_{xx}, ε_{yy} and γ_{xy} at an arbitrary point of the panel are related to the middle surface strains $\varepsilon_{x,0}$, $\varepsilon_{y,0}$ and $\gamma_{xy,0}$, and the changes in the curvature and torsion of the middle surface k_x, k_y and k_{xy} by the following three relationships:

$$\varepsilon_{xx} = \varepsilon_{x,0} + zk_x, \tag{1a}$$

$$\varepsilon_{yy} = \varepsilon_{y,0} + zk_y, \tag{1b}$$

$$\gamma_{xy} = \gamma_{xy,0} + zk_{xy}, \tag{1c}$$

where z is the distance of any arbitrary point on the panel from the middle surface. The strain components can be obtained in terms of displacements as follows: