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Jet with variable fluid properties: Free jet and dissipative jet

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ABSTRACT

An analysis of the laminar jet of an incompressible Newtonian fluid emerging from a narrow slot or a circular hole, where the physical properties like viscosity and thermal conductivity depends upon the temperature, is given. Both the cases: the case of In the absence of viscous heat dissipation and the case of In the presence of viscous heat dissipation are considered. The governing partial differential equations of the flow problem are transformed into the ordinary differential equations by group theoretic technique. The Runge–Kutta method is applied to obtained numerical solution of the transformed ordinary differential equations.

1. Introduction

When a fluid moving in a pipe or tank crosses a slot or a nozzle there is a sudden decrease in cross section area and consequently there is a considerable increase in velocity. It gives rise to the flow of a jet. A fluid flows in the absence of rigid boundaries and, therefore, it is a free flow. These flows are important in technology, metrology etc.

Fluid properties like viscosity, thermal conductivity cannot in general, be regarded as constant. In many physical situations such as liquid metal injectors and highly heated arcs etc. there is a large temperature difference between the temperature of jet and that of surrounding. Hence it becomes necessary to consider the variation of viscosity and thermal conductivity etc. with the temperature in case of jets as well. Density of a liquid changes with the temperature but the rate of decrease of density with increase of temperature is much smaller than that of viscosity and thermal conductivity. Therefore in all the analysis which follows density will be taken as constant while viscosity and thermal conductivity will be assumed to be temperature dependent. However, density variations when coupled with forces due to gravity will give rise to buoyancy forces. It will therefore, be desirable to take into account such forces in vertical jet.

A jet issuing from an orifice and mixing with a surrounding fluid at rest is a classical problem in jet flows. This has been discussed in most of the books on the fluid mechanics. In the study of jets one has to face some inherent difficulties; (i) satisfying homogeneous boundary condition at infinity, (ii) physical condition of constancy of flux of momentum or of heat or both.

The flow of a jet is divided into two main regions [1–3], the core and mixing region as shown in the Fig. 1. The core region is situated at the center of the jet and near the exit nozzle. In this region, viscous effects are negligible and the fluid may be considered to be inviscid. The mixing region begins at the edge of the core. In this zone, there is a large variation in velocity and density and the effect of viscosity and heat conduction must be considered.

The first study was recorded in literature by Schlichting [4] when he obtained closed form solution of laminar jet. Bickly [5], Squire [6] have discussed laminar jets of an incompressible fluid with different geometries of the orifice, Pai [7] studied two-dimensional and axisymmetric jet mixing of a compressible fluid. Kapur [8], Gutfinger et al. [9] introduced jets of a non-Newtonian fluid. In most of the above study it is assumed that the physical properties are constant with respect to temperature but it is valid only when the temperature differences are very small. The study of jet with variable fluid properties started in the year 1950. Ostrach [10] studied it in the case of flat plate problem. Extensive work in this area is carried out by many researchers. Among those it is worthy to quote here Spalding's et al. [11], Christiansen et al. [12] in flow through channels and tubes. Pai [7] suggested empirical relations for incompressible Newtonian and Non-

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Nomenclature

u, $\dot{u}$	Velocity components along X-axis
v, $\dot{v}$	Velocity components along Y-axis
ρ	Fluid density
μ	Viscosity
C_p	Specific heat at a constant pressure
K	Thermal conductivity
θ	Dimensionless temperature
M_0	Flux of momentum across any cross section perpendicular to the jet axis
Q	Heat flux across any cross section perpendicular to the jet axis
α	Fundamental length
υ	Kinematic viscosity
σ	Electrical conductivity
$f(\theta)$, $\phi(\theta)$	Temperature functions
Γ, δ	Some constants under the defined group transformation
η	Similarity variable
ψ	Stream function
$F(\eta)$, $G(\eta)$	Similarity functions

Newtonian fluid flow. A jet with variable fluid properties was simply analyzed by Kalathia [3]. Here, we shall solve it completely by the method of converting homogeneous Boundary value Problem (BVP) into non-homogeneous initial value problem. The detailed analysis of the problem and method of solution is discussed in this paper.

Jets are used in various industrial applications because of producing high heat and mass transfer coefficients. The flow structures of unconfined impinging jets are divided to three main regions: free jet region, stagnation region and wall jet region. The maximum Nusselt number is observed at the stagnation point or the surrounding point due to the formation of thin thermal boundary layer in that region. Confined impinging jets have these three global regions, but the upper wall that is in the same level with nozzle's exit restricts entrainment of jets, so the length of potential core in confined jets is more than free jets; therefore the Nusselt number has experienced greater amount related to free jet in the same condition for laminar impinging jets. Several investigations have been done on the impinging jets for many years that have been discussed the important factors on such flow structures. Chiriac and Ortega [13] has numerically studied the heat transfer, flow structures and transitional behaviors of confined slot jet at a fixed nozzle to surface distance and different Reynolds numbers. Park et al. [14] have numerically studied two dimensional laminar and turbulent impinging slot jets using *Simple*-base segregated streamline upwind Petrov- Galerkin finite element method. It has been indicated that this method is more accurate than other upwind numerical methods that used the artificial diffusion. Lee et al. [15] has numerically investigated the flow structures and distribution of the Nusselt number, and pressure coefficient along the flat plate. Lee et al. (2012) have experimentally measured the Nusselt number distribution over isothermal flat plate. This study focused on describing unsteady modes for each height ratio and Re number. Rady and Arquis [16] have numerically compared local and stagnation Nusselt number of a multiple impinging slot jets over an isothermal flat plate in laminar regime. They analyzed the effect of exhaust ports and spent flow on heat transfer rates, and they suggested that locating surface protrusions before exhaust ports can be enhanced the Nusselt number and reduce adverse effect of spent flow in multiple impinging jet systems. Kubacki and Dick [17] have numerically investigated the turbulent confined plane impinging jets using different turbulent models.

Kayansayan and Kucuka [20] both experimentally and numerically have investigated the flow structure and parameters on concave impinging plate. They analyzed turbulent and laminar regimes for confined plane impinging jets that are issued from the symmetry line of semi cylindrical channel. Measurement of pressure and Nusselt number distribution over impinging plate has been reported. They simulated the laminar flow by a numerical model too. It has been indicated the amount of the Nusselt number over target plate has been enhanced compared to flat plate due to surface curvature. Rahman and Hernandez [21] have numerically studied the conjugate heat transfer between a round water jet and convex surface. In this analysis, a constant heat flux is imposed on the bottom edge of the surface, and consideration of different solid material and plate thicknesses for the convex surface have been given generality to this investigation. Choi et al.

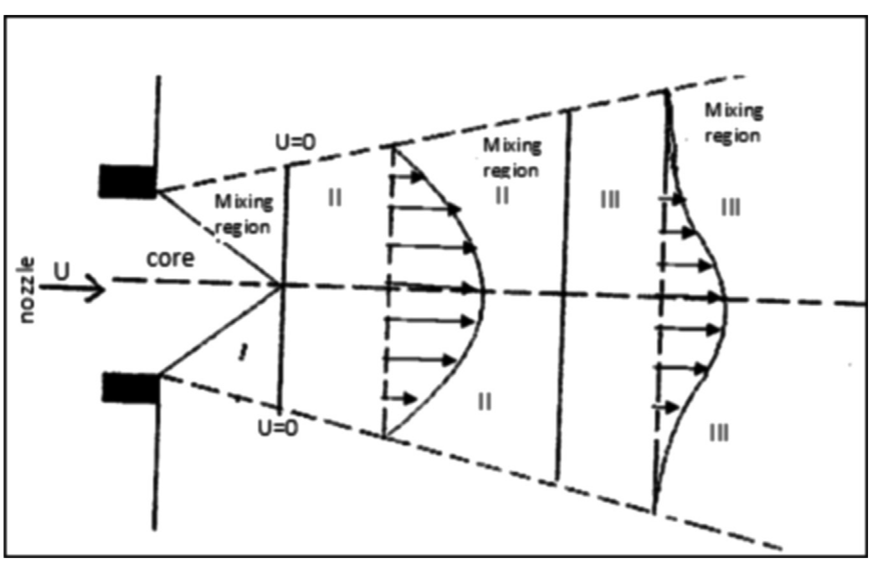

Fig. 1. Jet in a medium at rest.