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Identification of horseshoes chaos in a cable-stayed bridge subjected to randomly moving loads

L.M. Anague Tabejieu [a], B.R. Nana Nbendjo [a,b,*], U. Dorka [b]

[a] Laboratory of Modelling and Simulation in Engineering, Biomimetics and Prototypes, Faculty of Science, University of Yaounde I, P.O. Box 812, Yaounde, Cameroon

[b] Steel and Composite Structures, University of Kassel, Kurt-Wolters-Strasse 3, Kassel 34125, Germany

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ABSTRACT

In this paper, the dynamic response of cable-stayed bridge loaded by a train of moving forces with stochastic velocity is investigated. The cable-stayed bridge is modelled by Rayleigh beam with linear elastic supports. The stochastic Melnikov method is derived and the mean-square criterion is used to determine the effects of stochastic velocity and cables number on the threshold condition for the inhibition of smale horseshoes chaos in the system. The results indicate that the intensity of the random component of the loads velocity can be contributed to the enlargement of the possible chaotic domain of the system, and/or increases the chances to have a regular behavior of the system. On the other hand, the presence of cables in cable-stayed bridges system increases it degree of safety and paradoxically can be contributed to its destabilization. Numerical simulations of the governing equations are carried out to confirm the analytical prediction. The effect of loads number on the system response is also investigated.

1. Introduction

Cable-stayed bridges have become very popular over the last three decades because of their aesthetic appeal, structural efficiency, enhanced stiffness compared with suspension bridges, ease of construction and comparatively small size of structures. Response prediction of this type of bridges subjected to randomly moving excitations is important for engineering practice [1,2].

The vibrations of a suspension bridge under a random train of moving loads are discussed in detail by Bryja and Śniady [3–5]. Generally, a very important parameter in the study of the vibration of bridges caused by moving loads is the velocity. Although there is scarcity of publications on this subject, one can mention the work of Zibdeh [6] who included the effect of random velocities on the dynamic response of a bridge traversed by a concentrated load. Chang et al. [7] investigated the dynamic response of a fixed–fixed beam with an internal hinge on an elastic foundation, which is subjected to a moving mass oscillator with uncertain parameters such as random mass, stiffness, damping, velocity and acceleration. In the same impetus, Śniady et al. [8,9] and Rystwej et al. [10] investigated on the problem of a dynamic response of a beam and a plate to the passage of a train of random forces. In this study they assumed that the random train of forces idealizes the flow of vehicles having random weights and travelling at the stochastic velocity. They show the effect of these stochastic quantities on the mean deflection of the beam.

On one hand, in all of the above-mentioned research, only the effect of stochastic parameters of the moving loads on the probabilistic features of the beam response namely the mean square amplitude and the probability density function is carried out. To the best knowledge of the authors, the effects of stochastic fluctuations of the load velocity and the number of cables on the possible appearance of horseshoes chaos in the cable-stayed bridge system have not been explored by the researchers yet. Thus in this paper, based on the Melnikov approach, which is widely used by most researchers [11–15], all these effects on the appearance of transverse intersection of perturbed and unperturbed heteroclinic orbits and the route to chaos are investigated.

Following this introduction, the effective model of cable-stayed bridge is presented in Section 2. Also, the random Melnikov analysis for the examination of the effect of a noisy part of velocity of moving loads and cables effects on the threshold condition for the inhibition of chaos is extended. Section 3 presents some numerical simulations to validate the theoretical predictions. Finally, Section 4 is devoted to the conclusion.

* Corresponding author at: Laboratory of Modelling and Simulation in Engineering, Biomimetics and Prototypes, Faculty of Science, University of Yaounde I, P.O. Box 812, Yaounde, Cameroon.
E-mail address: rnana@uy1.uninet.cm (B.R. Nana Nbendjo).

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2. The bridge model

This section is devoted to the presentation of the system (Section 2.1), the corresponding reduced modal equations (Section 2.2). The last Section 2.3 is devoted to the theoretical analysis of the random Melnikov analysis applied to the proposed model.

2.1. Mathematical modelling

The dynamic model of a cable-stayed bridge system investigated in this paper and shown in Fig. 1(a) is the semi-harp type with two symmetrical spans. The cable-stayed bridge is modelled by using a Rayleigh beam theory [16] (in order to take into account the high frequency motion of the beam) of finite length L with geometric nonlinearities on elastic supports with linear stiffness K_i^c subjected to an axial compressive loads ${T_h}^c$ due to the total contribution of the horizontal component of the tensile cables and a series of lumped loads p moving along the beam in the same direction with the same stochastic velocity v_k (see Fig. 1 (b)). We assume that the mass of the cables is negligible and they are regularly spaced on the beam. Since all the stay cable anchorage sections are fixed to move both horizontally and vertically, the whole pylon is assumed to be fixed.

The deformed beam can be described by the transverse deflection $W = W(X, t)$ and the rotation of the cross section of the beam $\theta = \theta(X, t)$. By Considering the classical damping force model for the viscosity materials and Newton's second law of motion for an infinitesimal element of the beam, the equation of motion for the small deformations $\left(\theta\left(X, t\right) \simeq \frac{\partial W(X,t)}{\partial X}\right)$ for this system is then given by

$$m_b \frac{\partial^2 W}{\partial t^2} - R_a \frac{\partial^4 W}{\partial X^2 \partial t^2} + c\frac{\partial W}{\partial t} + T_h^c \frac{\partial^2 W}{\partial X^2} + \sum_{i=0}^{N_c} K_i^c \delta\left[X - i\frac{L}{N_c}\right] W + EI\frac{\partial^2}{\partial X^2}\left[\frac{\partial^2 W}{\partial X^2}\left(1 - \frac{3}{2}\left(\frac{\partial W}{\partial X}\right)^2\right)\right] = P\sum_{k=1}^{N_v} \varepsilon_k \delta\left[X - X_k(t - t_k)\right] \quad (1)$$

In which m_b, EI, ρ, R_a, c, $W(X, t)$ are the beam mass per unit length, the flexural rigidity of the beam, beam material density, the transverse Rayleigh beam coefficient, the damping coefficient and

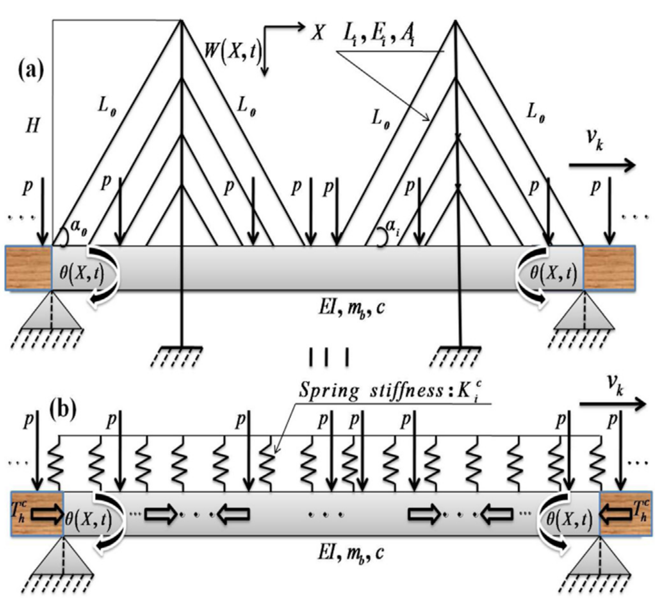

Fig. 1. Sketch of (a) the cable-stayed bridge system, (b) equivalent model under stochastic moving loads. The gravitational forces are represented by arrows p, whose separations are not uniform, for the speeds v_k are not identical.

the transverse defection of the beam at point X and time t respectively. ${T_h}^c$ is the axial compressive loads due to the total contribution of the horizontal component of the tensile cables. In Eq. (1), $m_b \frac{\partial^2 W}{\partial t^2}$ represents the inertia force of the beam per unit length, $R_a \frac{\partial^4 W}{\partial X^2 \partial t^2}$ is the rotary inertia force of the beam element (per unit length), $c\frac{\partial W}{\partial t}$ is the damping force of the beam per unit length, $T_h^c \frac{\partial^2 W}{\partial X^2}$ is the axial compressive load (per unit length) due to the horizontal component of the stay cables, $\sum_{i=0}^{N_c} K_i^c \delta\left[X - i\frac{L}{N_c}\right] W$ is the tension of cables per unit length, $EI\frac{\partial^2}{\partial X^2}\left[\frac{\partial^2 W}{\partial X^2}\left(1 - \frac{3}{2}\left(\frac{\partial W}{\partial X}\right)^2\right)\right]$ is the nonlinear rigidity of beam essentially due to the Euler law which states that the bending moment of the beam is proportional to the change in the curvature produced by the action of the load [17,18]. This nonlinear term is obtained by using the Taylor expansion of the exact formulation of the curvature up to the second order. The term on the right-hand side of Eq. (1) is used to describe the series of random moving loads over the beam. $X_k(t - t_k)$ is the distance covered by the kth force to the time t. $t_k = (k-1)d/v_0$ = deterministic arriving time of the kth load at the beam. d is the spacing loads, $\delta(\cdot)$ denotes the Dirac delta function, N_v is the total number of moving loads. To facilitate a compact representation of the equations, a window function ε_k is defined: $\varepsilon_k = 0$ when the load has left the beam and $\varepsilon_k = 1$ while the load is crossing the beam [19]. N_c is the number of cables acting on the bridge and $\delta[X - i\frac{L}{N_c}]$ give the position of each action. ${K_i}^c$ is the linear stiffness of the cables. Their expression according to the particular characteristics of the stay cables is given by [22]

$$K_i^c = \frac{E_i A_i}{L_i}\sin^2(\alpha_i) \quad (2)$$

where α_i is the angle between the ith cable and the bridge deck, E_i, A_i, L_i are Young's Modulus, the cross section and the length of the ith cable respectively. For a finite, simply supported beam, the boundary and initial conditions have the forms

$$W(X,t)\big|_{X=0,L} = 0, \quad \left.\frac{\partial^2 W(X,t)}{\partial X^2}\right|_{X=0,L} = 0.$$
$$W(X,t)\big|_{t=0} = 0, \quad \left.\frac{\partial W(X,t)}{\partial t}\right|_{t=0} = 0 \quad (3)$$

It is well known that a more realistic and practical model of highway traffic loads takes into account the features of the Poisson process [20], or the ones of renewal counting process [21] to represent the vehicular traffic. So to derive the proposed model of external forces, we take into account the randomness of the velocity and assume the similar form of loads studied by Nikkhoo et al. [19]. The random velocities are assumed to be Gaussian distributed, i.e. that the loads travel with velocities v_k Gaussian distributed around the average speed v_0 [8]

$$\frac{dX_k(t-t_k)}{dt} = v_k(t-t_k) = v_0 + \sigma_v \xi_k(t-t_k)$$
$$0 \le X_k(t-t_k) \le L \quad (4)$$

Here $v_k(t - t_k)$ is the stochastic velocity of the kth force, v_0 the mean value of velocity, σ_v its standard deviation and $\xi_k(t - t_k)$ the velocity disturbances which we assume to be independent and stationary white noise random processes; i.e.