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Dynamic stability in principal parametric resonance of rotating beams: Method of multiple scales versus differential quadrature method

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ABSTRACT

Parametric resonance is one of the key topics in studying the dynamics of structures. In this paper, dynamic analysis of rotating beams with varying rotational speed in the presence of principal parametric resonance is investigated. The equations of motion are based on the von Karman strain–displacement relationship. The beam is made of isotropic material with rectangular cross section. The flapping and axial motions are considered along the thickness and length of the beam, respectively. The Galerkin discretization approach is implemented to determine the natural frequencies. The method of multiple scales is applied directly to the ordered equations of motion for the dynamic stability analysis. The method of multiple scales delivers a closed form relation for the stability region boundary in terms of the adimensional rotational speed, axial mode frequency and damping ratio coefficient. The differential quadrature method is employed to validate the multiple scales results. A comprehensive study is accomplished to find the effects of damping ratio coefficient and mode number on the critical parametric excitation amplitude and the parametric excitation frequency. Damping ratio coefficient, mode number and parametric excitation amplitude influences on the stability region boundaries are also examined.

1. Introduction

Due to the vast implementation of rotating blades in various aerospace structures and also in the wind and water turbines, the dynamic analysis of these structures including the stability study is essential. To reduce the possibility of fatigue failure in these structures, studying the possibility of resonance occurrence including the parametric one is one of the concerns of the designing engineers.

The unwanted parametric resonance can occur in rotating beams with varying rotational speed. Variation in rotating speed can be produced by the aerodynamic forces. Discarding the source of the variation, if the ratio of the frequency variation of the rotational speed is proportional to one or any combination of the natural frequencies, parametric excitation can happen. On the other hand, large amplitude vibrations can be originated from parametric excitation, so the nature of this experience, i.e. parametric excitation can be risky to the structures.

When the excitation frequency is close to twice of a natural frequency, the first parametric resonance zone, i.e. principal parametric resonance occurs, while if the excitation frequency is close to any of the natural frequencies, the second parametric resonance zone, i.e. parametric excitation arises [1].

Crespo da Silva and Hodges [2] investigated the effects of higher order terms as well as aerodynamic forces on the instability of the coupled elastic flapping, lead-lagging, and torsional motions of the uniform straight rotating blades. Crespo da Silva [3] obtained the equilibrium solution and eigen-solutions of perturbed rotating beams via aerodynamic forces and investigated the stability of the structure about its equilibrium solution. Chin and Nayfeh [4] implemented the direct method of multiple scales (MMS) for nonlinear dynamic analysis and stability study of a clamped-hinged beam subjected to the principal parametric resonance. Ye et al. [5] studied the nonlinear dynamic behavior of a parametrically excited simply supported rectangular thin plate based on the von Karman strain–displacement relationships. The model considered symmetric cross-ply laminated composite plates and they also included the nonlinear damping effects. Using numerical method, the first-order averaged equations obtained by the MMS were analyzed to attain the steady state bifurcation responses. Chen and Yang [6] investigated the stability in transverse parametric vibration of axially accelerating viscoelastic beams. The

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MMS was applied directly to the governing equations of motion. The stability conditions were obtained for the combination parametric resonance and also principal parametric resonance. Ghayesh and Balar [7] studied the instability and bifurcation of axially moving Rayleigh beams, using MMS. De Rosa et al. [8] examined the dynamic behavior of a clamped beam subjected to a sub-tangential follower force at the free end. They determined the flutter critical load using differential quadrature method (DQM). Saravia et al. [9] examined the dynamic stability analysis of thin-walled composite rotating beams and the influences of the layup and rotational speed on the stability analysis. Turhan and Bulut [10] investigated the rotational speed effects on the nonlinear dynamics of a rotating beam, such as switching from hardening to softening and harmonic or super-harmonic jump phenomena by applying the Lindstedt–Poincaré method and the MMS to the equations of motion (EOM). Valverde and Garcia-Vallejo [11] studied on the stability of a rotating beam, in which the effects of Coriolis forces were considered in their formulation, by using the absolute nodal coordinate formulation in comparison with a fully geometrically exact nonlinear formulation based on the Cosserat theory of rods. Ghayesh [12] studied the parametric vibrations and the stability of an axially accelerating string guided by a non-linear elastic foundation using the MMS. Some numerical simulations were presented to highlight the effects of system parameters on vibration, natural frequencies, frequency-response curves, stability, and bifurcation points of the system. Ding and Chen [13] investigated the steady-state periodical response of an axially moving viscoelastic beam with hybrid supports via MMS and numerical confirmation by DQM. Numerical examples were presented to demonstrate the effects of the boundary constraint stiffness on the amplitude and the stability of the steady-state response. Arvin and Bakhtiari-Nejad [14] applied the MMS to the discretized EOM of rotating beams to construct the Nonlinear Normal Modes (NNMs) with or without internal resonances. They also studied the instability and bifurcations of the NNMs in the presence of internal resonances. Chen and Tang [15] investigated the parametric resonance stability region boundaries of axially accelerating viscoelastic beams using the MMS and DQM. Arvin et al. [16] examined the softening or hardening behavior of isotropic rotating beams mode frequencies and the rotational speed effects on the type of nonlinearity, either hardening or softening via MMS. The EOM were obtained [17] using the geometrically exact approach based on the Cosserat theory of rods. The similar study was conducted by Arvin and Lacarbonara [18] for composite rotating beams. Rhoads et al. [19] explored the highly non-linear dynamic behavior of a new class of parametrically excited, electromagnetically actuated micro cantilevers using the perturbation methods and the bifurcation analysis. The provided results clarified the effects of fifth-order non-linearities on a parametrically excited micro resonator. Franzini and Mazzilli [20] derived a unidirectional three-mode reduced-order model for the lateral motion of a slender and immersed rod subjected to harmonic and axial top motion. They concluded that, within the principal parametric instability region of the first mode, the time history corresponding to the second classic mode oscillates with the dominant frequency of the first classic mode. Awrejcewicz et al. [21] proposed a method to study dynamical instability and non-linear parametric vibrations of symmetrically laminated plates of complex shapes with different cutouts. In order to show the advantage of the developed approach, instability zones and response curves for the layered cross- and angle-ply plates with external cutouts were constructed and discussed.

In this paper, the stability analysis of rotating beams with varying rotational speed in the presence of principal parametric resonance is investigated. The EOM are based on von Karman strain–displacement relationship. The MMS is applied directly to the EOM subjected to the principal parametric resonance. The MMS renders a closed form relation for the stability region boundary and critical parametric excitation amplitude and frequency in terms of the adimensional parameters such as rotational speed, mode frequency and damping ratio coefficients. The DQM is also implemented to validate the MMS results. The results are presented in the form of tables and figures.

2. Equations of motion

A schema of a rigid rotating beam is presented in Fig. 1. R, h, b and L are the rotor radius, thickness, width and length of the beam, respectively. The beam is rotating about Z-axis with Ω as rotational speed. x parameter is used as the spatial variable which shows the position of each cross section measured from the cross section linked to the rotor.

The deformed shape of the beam is presented in Fig. 2. $\boldsymbol{u}$ is the deformation vector of a cross section at x which is $\boldsymbol{u} := \boldsymbol{u}(x, t) = w(x, t)\hat{z} + u(x, t)\hat{x}$ in which $w(x, t)$ and $\hat{z}$ are, respectively, the flapping deformation and the unit vector along the z-axis and $u(x, t)$ and $\hat{x}$ are, respectively, the axial deformation and the unit vector along the x-axis.

Assuming the von Karman strain displacement relations, the adimensional flapping and axial EOM and the associated boundary conditions for isotropic beams with rectangular cross section are, respectively, as [14]:

$$\partial_{tt}w + L^F(w) - n_2^F(u, w) - n_3^F(w, w, w) = 0 \tag{1}$$

$$B^i(w)\Big|_{x=0} = 0,\ B^{i+2}(w)\Big|_{x=1} = 0 \quad i = 0, 1 \tag{2}$$

and

$$\partial_{tt}u + L^A(u) - n_2^A(w, w) = 0 \tag{3}$$

$$B^0(u)\Big|_{x=0} = 0, \quad B^1(u)\Big|_{x=1} = n_B(w)\Big|_{x=1} \tag{4}$$

in which $L^F(\cdot)$ and $L^A(\cdot)$ are, respectively, the linear flapping and axial stiffness operators, $n_2^F(u, w)$, $n_2^A(w, w)$ and $n_3^F(w, w, w)$ are, respectively, the second and third order geometric nonlinear terms. $B^i(\cdot)$ and $n_B(\cdot)$ are the boundary conditions operators reported in Appendix A. The ∂ sign stands for the partial differentiation of the associated parameter with respect to its following

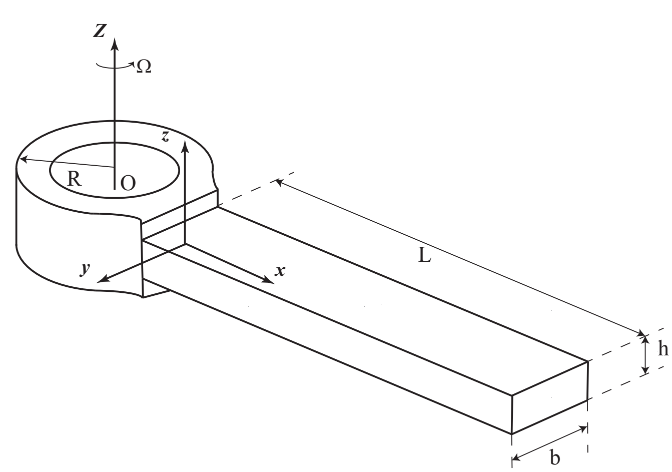

Fig. 1. A schema of the rigid rotating beam.