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Stochastic averaging of energy harvesting systems

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ABSTRACT

A stochastic averaging method is proposed for nonlinear energy harvesters subjected to external white Gaussian noise and parametric excitations. The Fokker–Planck–Kolmogorov equation of the coupled electromechanical system of energy harvesting is a three variables nonlinear parabolic partial differential equation whose exact stationary solutions are generally hard to find. In order to overcome difficulties in solving higher dimensional nonlinear partial differential equations, a transformation scheme is applied to decouple the electromechanical equations. The averaged Itô equations are derived via the standard stochastic averaging method, then the FPK equations of the decoupled system are obtained. The exact stationary solution of the averaged FPK equation is used to determine the probability densities of the displacement, the velocity, the amplitude, the joint probability densities of the displacement and velocity, and the power of the stationary response. The effects of the system parameters on the output power are examined. The approximate analytical outcomes are qualitatively and quantitatively supported by the Monte Carlo simulations.

1. Introduction

Energy harvesting from ambient kinetic energy to power low-powered electronics has emerged as a prominent research area and continues to grow rapidly. There are several excellent and comprehensive survey papers and monographs, notably Tang et al. [1], Pellegrini et al. [2], Harne and Wang [3], Daqaq et al. [4], Erturk and Inman [5] and Elvin and Erturk [6], reviewing the state of the art in different time phases of investigations related to energy harvesting.

Concentrating on the resonance under a harmonic excitation, most works on energy harvesting took the deterministic approach. Especially, nonlinearity has been introduced to increase the operating frequency range of energy harvesters. Mann and Sims [7] presented a novel energy harvesting device that used a magnet to produce a nonlinear restoring force, applied the method of multiple scales to characterize quantitatively the response, and validated the response amplitudes of numerical results by the experimental data. Mann and Owens [8] designed a nonlinear electromagnetic energy harvester that uses magnetic interactions to create a bistable potential well and validated the potential well escape phenomenon can be used to broaden the frequency response by theory and experiments. Erturk and Inman [9] constructed a piezomagnetoelastic energy harvester and investigated numerically and experimentally the response under harmonic excitation. Daqaq et al. [10] designed a nonlinear piezoelectric parametric excitation beam device, used the multiple scales method to analyze the response of the system subjected to harmonic excitation, and validated the analytically predicted response amplitudes with experiments. Zhu and Zu [11] presented a buckled-beam piezoelectric energy harvester that used a midpoint magnetic force to improve the efficiency of the output voltage. Zhou et al. [12] reported numerical and experimental investigations on a bistable piezomagnetoelastic energy harvester using rotatable magnets that enhanced broadband frequency response.

Since randomness inherent in most real-world circumstances may significantly change the behavior of energy harvesters, some researchers treated energy harvesting under random excitations via stochastic approaches. Cottone et al. [13] found numerically and experimentally that the nonlinear oscillators can outperform the linear ones under stochastic excitation. Based on a single-degree-of-freedom model, Adhikari et al. [14] analyzed the mean power of a linear piezoelectric energy harvester under stationary Gaussian white noise. Gammaitoni et al. [15] revealed that nonlinear oscillators can outperform the linear ones under Gaussian noise excitation in monostable configurations. However, Daqaq [16] demonstrated that monostable Duffing oscillator does not provide any enhanced power over the typical linear oscillators under Gaussian white noise and colored noise excitations. Litak et al. [17] calculated the response of a nonlinear piezomagnetoelastic energy harvester

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under stationary Gaussian white noise. Daqaq [18] derived an approximate expression for the mean power under exponentially correlated noise and demonstrated the existence of an optimal potential shape maximizing the output power. Analyzing a strongly nonlinear bistable piezomagnetoelastic energy harvester under random excitation, Ali et al. [19] established a closed-form approximate expression of the harvested power and validated against the Monte Carlo numerical simulation results. Green et al. [20] reported Duffing-type nonlinearities can reduce the size of electromagnetic energy harvesting devices without affecting their output power and verified the result using the technique of equivalent linearization. Jiang and Chen [21] used the Van Kampen expansion method to derive the statistical moments of the response from the FPK equation of piezoelectric coupling system and discussed the effects of the system parameters on the expected response moments. Masana and Daqaq [22] investigated the influence of stiffness-type nonlinearities on the transduction of buckling piezoelectric beam under band-limited noise by experiment. Daqaq [23] applied the method of moment differential equations of FPK equation to calculate response statistics and demonstrated that the energy harvester's time constant ratio plays a critical role in characterizing the performance of nonlinear harvesters in a random environment. Jiang and Chen [24] applied the equivalent linearization technique to calculate response of Duffing-type nonlinear piezoelectric energy harvester under Gaussian white noise excitation and demonstrated that the equivalent linearization approximated the original nonlinear system well for the weak nonlinearity.

Among various approaches nonlinear random vibration, the stochastic averaging method is a powerful approximate technique for the prediction of response of nonlinear system subject to external and parametric random excitations. The success of the stochastic averaging method is mainly due to the reduction of dimensions of the FPK equation while the essential behavior of the system is retained. It is a convenient approximation approach to predict the stationary response of nonlinear stochastic systems and has been extensively used in theory and engineering application of random vibration. There are several excellent and comprehensive survey papers, notably Roberts [25], Crandall and Zhu [26] and Zhu [27], reviewing the state of the art in different time phases of investigations related to stochastic averaging methods in random vibration. Historically, Roberts [28] employed the stochastic averaging method to investigate the response of ship rolling motion and obtained the exact stationary probability density function. Roberts and Spanos [29] applied the stochastic averaging method to study the response of nonlinear oscillator under external excitation with or without combined parametric excitation, and obtained the exact stationary and non-stationary probability density function. In recent years, Huang and Zhu [30] developed the stochastic averaging method for Hamiltonian systems and obtained many important results. So far, to the authors' best knowledge, there is no stochastic averaging analysis on energy harvesting. To address the lack of research in this aspect, the present work develops the stochastic averaging technique to determine the response of nonlinear energy harvesters under Gaussian white noise excitation.

The paper is organized as follows. Section 2 treats a snap-through electromagnetic energy harvester. Section 3 treats a nonlinear piezoelectric energy harvester. Section 4 treats a parametric random excitation energy harvester. Section 5 contains the concluding remarks.

2. Stochastic averaging of a nonlinear electromagnetic energy harvesting system

It should be remarked that there are two different types of electrical circuit equations used in electromagnetic energy harvesting. Mann and Sims [7], Daqaq [16,18] and Green et al. [20] proposed a uncoupled electromechanical equation for lumped-parameter energy harvesters which neglected the inductance. Mann and Owens [8] and Li and Xiong [31] presented a coupled electromechanical equation for lumped-parameter energy harvesters which accounted for the inductance.

The nonlinearity will be produced by a snap-through mechanism. It consists of two inclined linear springs connected to a damper and a mass. In spite of the linearity of each spring, the geometrical configuration results in a nonlinear restoring force. It is a convenient way to exhibit nonlinear behaviors. Although the snap-through mechanism is well studied, the works on its application in energy harvesting are rather limited. Li and Xiong [31] introduced the mechanical-electrical conversion as an electromagnetic generator and analyzed numerically its nonlinear behavior under periodic excitations. Jiang and Chen [32] employed the snap-through mechanism to harvest electricity from random vibration through piezoelectricity and revealed numerically that the snap-through energy harvester can outperform the linear energy harvester in the similar size under Gaussian white noise. This section applies the standard stochastic averaging method to an electromagnetic energy harvester based on the snap-through mechanism under Gaussian white noise excitation.

Fig. 1 shows the lumped-parameter model of an electromagnetic energy harvester based on the snap-through mechanism. The governing coupling equation under Gaussian white noise excitation can be written as

$$m\ddot{x} + c\dot{x} + 2k\left(1 - \frac{L}{\sqrt{x^2 + l^2}}\right)x + mg + BIL_{coil} = m\xi(t), \tag{1}$$

$$L_{ind}\dot{I} + RI - BL_{coil}\dot{x} = 0 \tag{2}$$

where m is the mass, x is the displacement response, c is a mechanical damping term, k is stiffness of spring, L is the original length of the spring, l is the distance between the center and the edge of the frame and ϕ is the inclination of the spring with respect to the origin, g is the gravitational acceleration. The

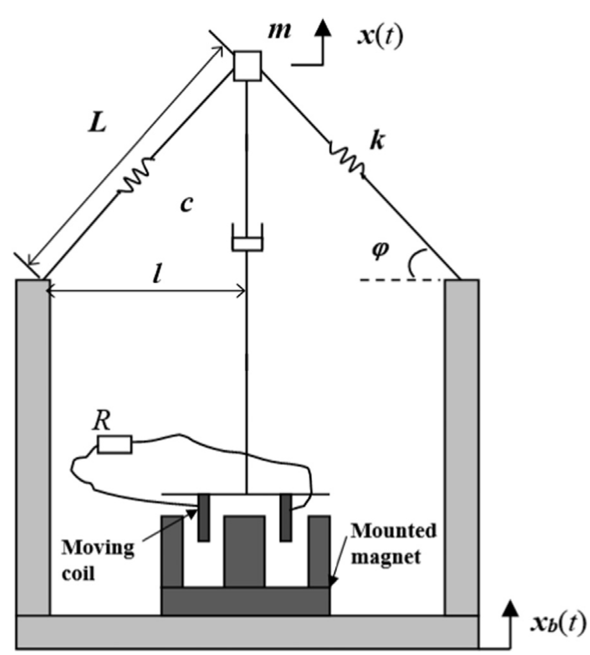

Fig. 1. Schematic of the electromagnetic energy harvester.