International Journal of Non-Linear Mechanics 85 (2016) 226–240

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Damping for large-amplitude vibrations of plates and curved panels, part 2: Identification and comparisons

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ARTICLE INFO

Article history:
Received 9 November 2015
Received in revised form
14 May 2016
Accepted 16 May 2016
Available online 18 May 2016

Keywords:
Non-linear identification
Rectangular plates
Curved panels
Non-linear vibrations
Damping
Experiments

ABSTRACT

A non-linear identification technique based on the harmonic balance method is presented to obtain the damping ratio and non-linear parameters of isotropic and laminated sandwich rectangular plates and curved panels, subjected to harmonic excitation orthogonal to the surface. The response of structures under consideration is approximated by a single-degree of freedom Duffing oscillator accounting for viscous damping, quadratic and cubic non-linear stiffness. The method uses experimental frequency-amplitude data and a least-squares technique to identify parameters and reconstruct frequency-response curves by spanning the excitation frequency in the neighborhood of the lowest natural frequencies. In particular, an iterative procedure is implemented to construct the mean displacement and identify the damping ratio. Close agreement is seen between the reconstructed non-linear frequency-amplitude curves, the experimental data and the results of the reduced-order model obtained in part 1 of the present study (Alijani et al., 2015 [1]). The proposed identification technique confirms the very large increase of damping during large-amplitude vibrations, as observed in part 1 of the present study, and demonstrates a non-linear correlation between damping, vibration amplitude and excitation level.

1. Introduction

In part 1 of the present study [1], a model was developed and experiments were conducted to investigate geometrically non-linear forced vibrations of plates and panels. In particular, five different sets of experiments were carried out on (i) plates and curved panels made of stainless steel having supported and free boundary conditions and (ii) Carbon/Epoxy laminated sandwich plates with foam and honeycomb cores. This part 2 of the present study presents an identification procedure and the identified damping as a function of the vibration amplitude. The identification procedure has the dual aim of (i) tracking the evolution of damping with the vibration amplitude; (ii) estimating the geometric non-linearities that are present in the vibration response of plates and curved panels.

Non-linear system identification has received considerable research attention in recent years due to the importance of obtaining accurate numerical models that predict the response of structures during large-amplitude vibrations. These identification techniques are classified into several categories, including time-domain and frequency-domain methods. A comprehensive treatment of non-linear system identification, in both frequency and time domain, is presented in the monograph of Warden and Tomlinson [2]. A survey of the non-linear system identification methods could also be found in Billings [3] and Kreschen et al. [4]. In particular, the review article by Kreschen et al. [4] elaborates seven different non-linear system identification categories available to date.

The most commonly used time-domain methods are the restoring-force surface or force-state mapping method [5,6] that uses Chebyshev polynomials for expanding non-linear restoring forces. Other time domain methods include those based on non-linear auto-regressive moving average models with exogenous inputs (NARMAX) [7], Hilbert transform [8,9] and Lie series solution [10]. Comparing to frequency domain methods, time domain methods require less effort for data acquisition and processing. However, they face problems in differentiating noisy signals. On the other hand, frequency domain methods avoid the problems associated with temporal data, but require more in depth theoretical effort. Early attempts in frequency domain identification were based on Volterra and Wiener series [2,3]. The Method of Multiple Scales (MMS) and harmonic balance are frequently used for performing non-linear identification in frequency domain. For instance, Krauss and Nayfeh [11] used the Amplitude and Frequency-Sweep Method (AFSM) together with MMS to perform experimental non-linear identification on single-mode transversely excited beams.

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http://dx.doi.org/10.1016/j.ijnonlinmec.2016.05.004
0020-7462/

Malatkar and Nayfeh [12] used the peak of the non-linear frequency response curves and MMS to estimate the modal parameters of a steel beam subjected to base excitation considering quadratic damping. The harmonic balance method was used by Yasuda et al. [13] in an inverse way to perform system identification in multi-degree of freedom lumped systems. Parametric identification of an experimental magneto-elastic oscillator using harmonic balance method was carried out by Feeny et al. [14]. Thothadrai et al. [15] and Thothadrai and Moon [16] used harmonic balance to conduct multi-degree of freedom system identification in experiments dealing with self-excited motions and fluid-structure interactions.

A challenging concept in non-linear system identification is the identification of damping from experimental data. Dissipation is intrinsically a non-linear phenomenon during large-amplitude vibrations and it is not yet well-established. The modal damping assumption is a convenient tool that has been extensively used to model dissipation. However, it is not yet proven that this model could give an insight of the physical reality of the damping behavior. That's why, different non-linear damping mechanisms have been proposed with the most common being the quadratic damping of the form $c\dot{x}|\dot{x}|$ (c being the damping coefficient and $\dot{x}$ the velocity) where the absolute value ensures that the damping force is always opposed to the velocity [2]. Other forms of non-linear damping include quadratic and cubic powers of relative velocity and hysteresis. Mei and Prasad [17] used a non-linear damping model comprising of quadratic displacement multiplied by the velocity ($cx^2\dot{x}$) to investigate the damping response of beams under acoustic loading. Mahmoodi et al. [18] proposed a general solution based on MMS to study non-linear vibrations of damped continuous systems. In their study, damping was assumed to be a combination of $c_1\dot{x}^3 + c_2x^2\dot{x}$. Recently, a similar damping model was used by Ozcelik and Attar [19] to study the effect of non-linear damping on the dynamics of flapping beams. The concept of the cubic damping ($c\dot{x}^3$) has also been used by Ye et al. [20] to study the chaotic behavior of composite rectangular plates under parametric excitation. Non-linear damping mechanisms due to hysteresis can also be found in Caughey and Vijayaraghavan [21] and Al-Bendar et al. [22].

Another damping model that is quite often used is the viscoelastic model, which is based on rheological models such as the Kelvin–Voigt, Maxwell or their generalized versions [23]. Among works that have investigated non-linear vibrations of viscoelastic plates and panels, one could refer to Esmailzadeh and Jalali [24], Touti and Cederbaum [25], Billasse et al. [26] and Mahmoudkhani et al. [27]. Recently, Amabili [28] investigated large amplitude vibrations of Kelvin–Voigt imperfect plates via multi-mode Lagrangian approach.

Although there have been numerous studies concerning non-linear damping, so far none have discussed the non-linear correlation between the damping, vibration amplitude and the excitation force level during large-amplitude excitations. Therefore, different from previous studies, in this paper, a non-linear identification technique based on harmonic balance method is presented to examine the damping behavior of plates and panels studied experimentally in part 1 [1]. The identification technique gives also an estimate of the strength of non-linearity by identifying the non-linear parameters assuming that the response of the system is approximated by a single Duffing oscillator with viscous damping, quadratic and cubic non-linearities. Here it should be noted that, even though viscous damping has been considered to model the damping behavior, this model could indeed give a general perspective of how damping varies non-linearly with the vibration amplitude since the damping ratio is estimated at each single excitation level. The identifications are performed on: (i) a stainless steel rectangular plate with four free edges; (ii) a sandwich rectangular plate with Carbon/Epoxy skins having (0/90) stacking sequence and a DIAB® Divinycell foam core with free edges; (iii) a second sandwich plate with Carbon/Epoxy skins having (0/90) lay-up and a PLASCORE® PN2 aramid fiber honeycomb paper core with free edges; (iv) a stainless steel rectangular plate with supported edges; (v) a circular cylindrical stainless steel panel with simply supported boundary conditions. The frequency-amplitude data obtained from experiments are used as the inputs for the identification scheme and the least squares method is utilized to minimize the error between the measured response and the identified model. Moreover, an iterative algorithm is implemented to extract the mean displacement (DC component) from the experimental data and obtain the damping ratio with accuracy. It is observed that damping grows non-linearly with the increase in the vibration amplitude. Furthermore, the extracted non-linear parameters show that the experimentally tested plates and panels are weakly non-linear systems. Finally, comparisons are performed between the identified curves, the experimental data and the reduced-order models developed in part 1 of the present study [1] showing very good agreements. Therefore a double damping identification is performed at each excitation level, based on the single Duffing equation proposed here and the full non-linear plate/panel model proposed in [1], and results are compared.

2. Non-linear identification method

2.1. Harmonic balance method

A classical example of a non-linear system, originally studied by Duffing, is the forced mass–spring system with viscous damping, where the restoring force of the spring is non-linear (see Fig. 1). The equation of motion for this system is:

$$m\ddot{x} + c\dot{x} + k_1x + k_2x^2 + k_3x^3 = f\cos(\Omega t), \tag{1}$$

where m is the mass, c is the viscous damping coefficient, k_1 is the linear stiffness, k_2 the quadratic stiffness, and k_3 the cubic stiffness. Moreover, x is the vibration amplitude, f is the force excitation, Ω is the excitation frequency and t is time. This system has been extensively used for studying non-linear vibrations of structures subjected to external harmonic excitation around the frequency of the fundamental mode. Eq. (1) can be effective if the fundamental mode of vibration is not involved in an internal resonance with other modes. If such condition retains, then other modes accidentally excited, will decay with time to zero due to the presence of damping [29]. In this paper, it is assumed that this condition is preserved and therefore the response of the plates and panels studied in part 1 [1] are described by a single Duffing oscillator for performing non-linear identification.

By assuming that the plate/panel has thickness h, one could make Eq. (1) dimensionless as follows:

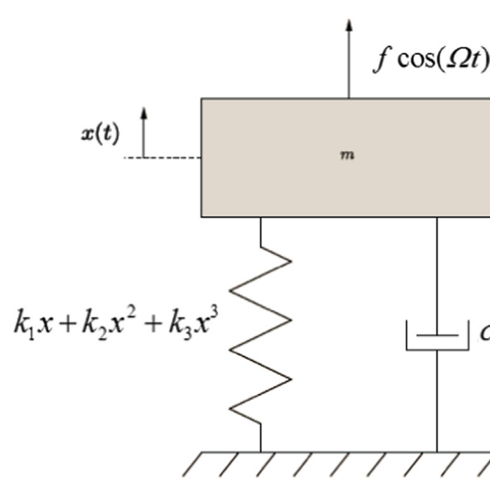

Fig. 1. The single-degree-of-freedom model.