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Forward and backward motion control of a vibro-impact capsule system

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ABSTRACT

A capsule system driven by a harmonic force applied to its inner mass is considered in this study. Four various friction models are employed to describe motion of the capsule in different environments taking into account Coulomb friction, viscous damping, Stribeck effect, pre-sliding, and frictional memory. The non-linear dynamics analysis has been conducted to identify the optimal amplitude and frequency of the applied force in order to achieve the motion in the required direction and to maximize its speed. In addition, a position feedback control method suitable for dealing with chaos control and coexisting attractors is applied for enhancing the desirable forward and backward capsule motion. The evolution of basins of attraction under control gain variation is presented and it is shown that the basin of the desired attractors could be significantly enlarged by slight adjustment of the control gain improving the probability of reaching such an attractor.

1. Introduction

In the last few years, there was a growing interest in developing mobile mechanisms for minimally invasive surgical operation [1–4] and engineering pipeline inspection [5–7]. Particularly, investigation of a capsule system moving under internal force when overcoming environmental resistance has attracted significant attention, e.g. [8–11]. The merit of such a system is its simplicity in mechanical design and control which does not require any external driving mechanisms while allows it to move independently in a complex environment unaccessible to the legged and wheeled mechanisms [12,13]. However, any small uncertainties in friction or system parameters may lead to qualitative change of the dynamics of the capsule system [14]. Therefore understanding of the dynamics and motion control under different frictional environments for such a system is essential.

This paper studies the vibro-impact dynamics of a capsule system in the environments described by four friction models under variation of the amplitude and the frequency of harmonic excitation. The physical model of the vibro-impact capsule system is shown in Fig. 1 which consists of a capsule main body interacting with an internal harmonically driven mass. An initial bifurcation study of this system was carried out in [11]. The study has shown that the dynamic behaviour of the system is mainly periodic, and the best progression can be achieved through a careful choice of system parameters, such as mass ratio, stiffness ratio, amplitude and frequency of excitation. The dynamics of the capsule system in various frictional environments under variation of the mass ratio was investigated in [14] which suggested that directional control of the system can be achieved either by varying its mass ratio or by switching between coexisting attractors. This paper proposes a position feedback control law in order to control the capsule moving along a desired direction. Additionally, we also show that our proposed control method is capable for the control of chaos or coexisting attractors for ensuring an efficient performance of the system.

Control of vibro-impact systems has attracted great attention for many years, e.g. [15–20]. In [15], control of a double impacting oscillator using displacement feedback was studied, and the effect of how grazing impacts limit the stability regions of certain periodic orbits was discussed. To retain the existence of a desired attractor near the grazing trajectory, Dankowicz and Jerrelind [16] employed a discrete linear feedback control strategy. In [17], Souza and Caldas introduced a transcendental map to determine the value of parameter perturbation for controlling the vibro-impact systems which exhibited desired unstable periodic orbit embedded in a chaotic attractor. Lee and Yan studied the control algorithms for position control of an impact oscillator and synchronization of two impact

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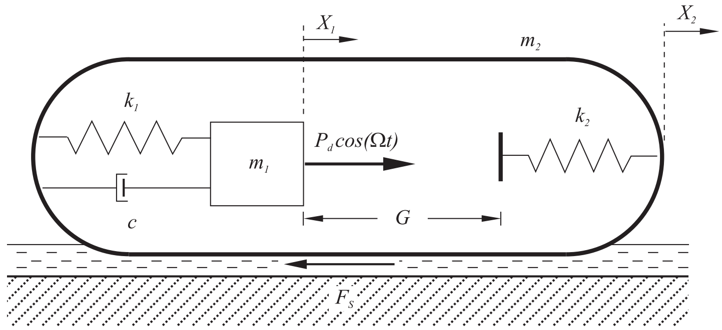

Fig. 1. Physical model of the vibro-impact capsule system.

oscillators in [18]. A feedback control technique using a small-amplitude damping signal was studied in [19] for suppressing chaotic behaviour of an impact oscillator. Later on, Wang et al. [20] developed an impulsive control method to stabilize the chaotic motion for a class of vibro-impact systems. In [21], Liu et al. proposed an intermittent control method for a class of non-autonomous dynamics systems that naturally exhibited coexisting attractors, and demonstrated its applicability to an impact oscillator numerically and experimentally. Basically, vibro-impact systems mainly fall into two categories: the impact oscillator with fixed impact body, e.g. [22,23] and the impact oscillator with one-side drifting impact body e.g. [24–26]. However this paper studies the control of a vibro-impact system with impact body drifting forward and backward which has never been considered in the literature before. Compared to the impact oscillator with fixed impact body, this structure induces more complicated dynamics when experiences various environmental resistance. Both optimization and directional control issues must be considered for our proposed system, while only optimization is needed for the impact oscillator with one-side drifting impact body. This paper also uses basins of attraction for the first time to investigate the possibility of switching between coexisting attractors by using the proposed control method.

The rest of this paper is organized as follows. In Section 2, mathematical modelling of the vibro-impact capsule system is presented, and the four different friction models used in this paper are briefly introduced. In Section 3, a non-linear dynamic analysis of the capsule system is conducted by varying the amplitude of excitation. In Section 4, influence of excitation frequency on capsule dynamics is investigated, and its global and local optima are studied. In Section 5, forward and backward motion control of the capsule system is studied by using a position feedback control law. Here the capabilities of the proposed control method for the control of chaos and coexisting attractors are demonstrated through extensive numerical studies. Finally, some concluding remarks are drawn in Section 6.

2. Mathematical modelling

2.1. Equations of motion

This work considers a two degrees-of-freedom dynamical system depicted in Fig. 1, where a movable internal mass m_1 is driven by a harmonic force with amplitude P_d and frequency Ω interacting with a rigid capsule m_2 via a linear spring with stiffness k_1 and a viscous damper with damping coefficient c. X_1 and X_2 represent the absolute displacements of the internal mass and the capsule, respectively. The internal mass contacts a weightless plate connected to the capsule by a secondary linear spring with stiffness k_2 when the relative displacement X_1-X_2 is larger or equals to the gap G. When the force acting on the capsule exceeds the threshold of the dry friction force F_b between the capsule and the supporting environmental surface, the bidirectional motion of the capsule occurs, and the friction force F_s is applied to the capsule.

To simplify the analysis, we introduce the following non-dimensional variables:

$$\tau=\Omega_0 t,\quad x_i=\frac{k_1}{P_f}X_i,\quad y_i=\frac{dx_i}{d\tau}=\frac{k_1}{\Omega_0 P_f}\dot{X}_i,\quad \dot{y}_i=\frac{dy_i}{d\tau}=\frac{k_1}{\Omega_0^2 P_f}\ddot{X}_i,$$
$$f_s=\frac{F_s}{P_f},\quad f_b=\frac{F_b}{P_f},$$

and parameters

$$\Omega_0=\sqrt{\frac{k_1}{m_1}},\quad \omega=\frac{\Omega}{\Omega_0},\quad \alpha=\frac{P_d}{P_f},\quad \xi=\frac{c}{2m_1\Omega_0},\quad \delta=\frac{k_1}{P_f}G,\quad \beta=\frac{k_2}{k_1},$$
$$\gamma=\frac{m_2}{m_1},$$

where $i=1$, 2, and P_f is the threshold of Coulomb friction. The considered system operates in bidirectional stick-slip phases which contain the following modes: *stationary capsule without contact, moving capsule without contact, stationary capsule with contact, moving capsule with contact*. A detailed consideration of these modes and dimensional form of the equations of motion can be found in [11]. The comprehensive equations of motion for the vibro-impact capsule system are written as

$$\begin{aligned}
\dot{x}_1&=y_1,\\
\dot{y}_1&=\alpha\cos(\omega\tau)+(x_2-x_1)+2\xi(y_2-y_1)-H_3\beta(x_1-x_2-\delta),\\
\dot{x}_2&=y_2(H_1(1-H_3)+H_2H_3),\\
\dot{y}_2&=(H_1(1-H_3)+H_2H_3)\big(-f_s-(x_2-x_1)-2\xi(y_2-y_1)\\
&\quad+H_3\beta(x_1-x_2-\delta)\big)/\gamma,
\end{aligned}\tag{1}$$

where $H(\cdot)$ is the Heaviside function and functions H_i $(i=1,2,3)$ are defined as

$$\begin{aligned}
H_1&=H(|(x_2-x_1)+2\xi(y_2-y_1)|-f_b),\\
H_2&=H(|(x_2-x_1)+2\xi(y_2-y_1)-\beta(x_1-x_2-\delta)|-f_b),\\
H_3&=H(x_1-x_2-\delta).
\end{aligned}$$

2.2. Friction models

In [14], the environmental resistance was described by four different friction models given in Table 1 which took into account Coulomb friction, viscous damping, Stribeck effect, pre-sliding, and frictional memory. As it is known, the Coulomb friction model provides the first approximation of dry frictional contact, and the Coulomb viscous damping model takes into account the viscosity of lubricated contact. Both Coulomb Stribeck and seven-parameter models [27] describe the friction of thicker lubricated contact, while the later one can comprehensively interpret the resistant force at a very low relative speed. The work in [14] has revealed that when the weight of the internal mass is smaller than the weight of the capsule, the nature of the friction mechanism