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# Rotating a pendulum with an electromechanical excitation

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ABSTRACT

The objective of this paper is showing investigation of pendulum rotations via vertical, non-linear electromechanical excitation generated using a RLC-circuit-powered solenoid, which is originally built for an electro-vibro-impact mechanism. Various non-linear phenomena of pendulum dynamics, namely period-1 rotation, period-1 oscillation and period-2 oscillation, have been observed experimentally from the proposed apparatus. A mathematical model has been developed for the experimental rig and the system parameters have also been identified for the mathematical model. Finally, numerical results have been generated using the developed mathematical model and identified parameters, and their correlations with experimental observations have been discussed.



## 1. Introduction

A parametric pendulum model is a kind of forced pendulum whose pivot is subjected to an alternating vertical motion, which is a time dependent parameter. The parametric pendulum model, although its mathematical expression is simple and comprehensible, is inherent with phenomena of bifurcations and chaos, and these have made it a rich area for research. Applications of parametric pendulum model include, but not limited to, mechanics, electronics, non-linear optics, solid state physics, chemical kinetic, biology, population dynamics [1] and, recently, renewable energy [2].

Extensive investigations were performed in the past decades in the research area of non-linear dynamics to quantify the non-linear characteristic and dynamic responses of the parametric pendulum model. While attention was focused on investigating non-linear behaviors of the pendulum (e.g. [3–8]), studies on the rotating solutions in parameter space of the parametrically excited pendulum model have flourished in recent years. For example, harmonic and subharmonic rotating solutions were discovered upon computation of bifurcation of systems by applying Melnikov's method in [1]. Global bifurcation, fractal structure of basin boundaries and co-existence of rotating solutions were investigated numerically in [9,10]. Solutions of steady-state stable rotation of the parametrically excited pendulum were classified in [11,12]. The stability of the rotational orbits within the resonance tongues in plots of parameter space was explored numerically in [13]. Rotating solution was obtained analytically in [14–16]. The influence of stochastic excitation on rotation of the parametric pendulum was investigated in [17]. Attractor robustness and basin integrity of rotation of the parametric pendulum model were discussed in [18]. The parametric pendulum model was subjected to elliptical excitation and its influences to the stable rotation region in the parameter space were investigated numerically and analytically in [19] and [20], respectively. Apart from theoretical analysis, early experiment work was performed in [4,5] to investigate experimental bifurcation of the parametrically excited pendulum. Experimental studies on interactions between parametric pendulum and electro-dynamical shaker were reported in [21]. An experimental investigation was carried out in [22,23] with the aim of exploring the response of pendulum to the wave forcing in a dedicated wave flume laboratory. The basin of attraction for periodic rotating orbits, which had been foreseen in the parameter space in [1,16], was reported in [22] to be smaller than the theoretical one in experiment, implying that the operating regime of parametric pendulum systems that yield stable rotation is expected to be narrower in real life than theory. To address this shortcoming, delayed-feedback control, proposed in [24], was introduced in [25] with the aim of initiating the rotational motion regardless of the initial conditions and avoiding bifurcations that destabilize the rotation motion of pendulum while maintaining stable rotation. It was used for start up control of the pendulum in [26], and the tolerance of start up control and delayed feedback control for maintaining periodic rotation was investigated in [27,28]. Its variant control strategy, extended time-delayed feedback which was proposed in [29], was used in [30] to control bifurcation for the parametric pendulum under harmonic excitation.

The motivation of this study is the idea of rotating a pendulum experimentally by implementing vertical actuation of the parametric pendulum system using a linear actuator. The linear actuator is in the form of solenoid, which is powered by a solid-state-relay (SSR)-coupled RLC circuit. A metal core is situated

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inside the solenoid and the metal core is mounted to pivot point of pendulum by means of mechanical linkages. Electromechanical excitation is generated within the solenoid onto the metal core, and the whole assembly is oscillated vertically under the influence of electromagnetic force and gravity. The input of the system is in the form of square waves and a tension spring is used in the proposed setup to compliment the vertical oscillation of the parametric pendulum model. The type of parametric pendulum system considered in present work belongs to the special class of autoparametric systems; the pendulum is excited as it experiences autoparametric resonance with the electromechanically-excited mass-spring-damper system [31]. A similar type of parametric pendulum system was reported in [32], where the non-linear phenomena of pendulum suspended on the forced Duffing oscillator were studied numerically. The behavior of parametric pendulum pivoted on the mass–spring–damper system was also investigated analytically, numerically and experimentally around the principal resonance region in [33]. In present work, the vertical actuation of parametric pendulum is performed by switching the SSR-coupled RLC circuit on and off via a train of square pulses, unlike the ideal harmonic forcing cases in [32,33]. This type of parametric pendulum system is relatively new and it has not been reported in the open literature.

The paper is organized in the following manner. The new experimental setup is first described in Section 2. The physical model and mathematical modeling are presented in Section 3. Some preliminary results on correlation between the experiment rig and the developed mathematical model are presented in Section 4. Finally, the paper is summarized and concluded in Section 5.

## 2. Experimental rig

The setup of parametric pendulum system that facilitates the investigation of pendulum dynamics subjected to non-linear electromechanical excitation is depicted in Fig. 1. The solenoid is mounted in series with a linear bearing on an aluminium base plate, which is clamped firmly on a rigid aluminium frame using a locking plate. The aluminium frame, in turn, is clamped on to the table firmly using a F-clamp. The conductor, a steel bar, is sandwiched in between two brass bars with different diameters. The upper brass bar is connected to a tension spring and it is inserted into the linear bearing to ensure that the assembly is oscillated only in the vertical direction and eliminates the pitch motion. The tension spring is in turn suspended by means of a mechanical hook and there are threaded holes on the aluminium base plate which allows the suspension point to be adjusted along the vertical direction. On the other hand, the lower brass bar connects the conductor to the cart assembly. The cart assembly pivots the pendulum with bob and houses the transducers of the system. An incremental rotary encoder is mounted in series with the pendulum pivot point using plastic couplings. On the other hand, an accelerometer is mounted by means of cap screws on the base plate of the cart assembly. A pair of acrylic sheets is used as a vertical guide to constrain the cart to move only in one direction while minimizing the yaw motion caused by the coupling motion from the pendulum.

Fig. 2 illustrates the schematic of the experimental setup. From this figure, the pendulum assembly is oscillated vertically by means of exerting electromagnetic force periodically on the metal bar within a solenoid. Such forcing requirement could be fulfilled by connecting the solenoid in series with a capacitor and an external power supply; a RLC circuit in a similar way to Mendrela and Pudlowski [34], as depicted in Fig. 2. A solid state relay, which has an input signal generated from a signal generator, is coupled with the circuit to switch the solenoid on and off and thus producing the periodic forcing effect. A tension spring is used to complement the vertical oscillatory motion of pendulum assembly while preventing the assembly from free falling under the influence of gravitational force. The solenoid used in present work was previously employed in the research work of electro-vibro-impact system in [35]. The specifications of the solenoid are adopted from [35] as follows. It is a custom-made solenoid that has 2400 turns of 0.56 mm diameter isolated wire winding around a PVC spool, and the spool has inner diameter and thickness of 28 mm and 2 mm. The diameter of the coil is 65 mm and it covers 60 mm of the length of the spool, and they are contained in a cylindrical steel frame as depicted in Fig. 1. To adapt the influence of gravitational force in the vertical configuration, the equilibrium position of the pendulum assembly is adjusted in the way such that the center of the metal conductor is located inside but at some distance below the center of the solenoid. When the RLC circuit is switched on via the solid state relay, the metal conductor is pulled towards the upper extreme end of the solenoid due to strong magnetic generated by a resonant current. As the metal conductor is rigidly connected to the pendulum assembly via a brass cylinder, the generated electromagnetic force also accelerates the pendulum assembly upwards while overcoming its inertia, gravitational force, compression force of tension spring and frictional force. When the RLC circuit is switched off, the magnetic field disappears and the pendulum assembly is accelerated downwards under the influence of gravitational force while overcoming its inertia, tension force of tension spring and frictional force. This in turn moves the metal conductor back to the lower extreme end of solenoid. By repeating the above processes,

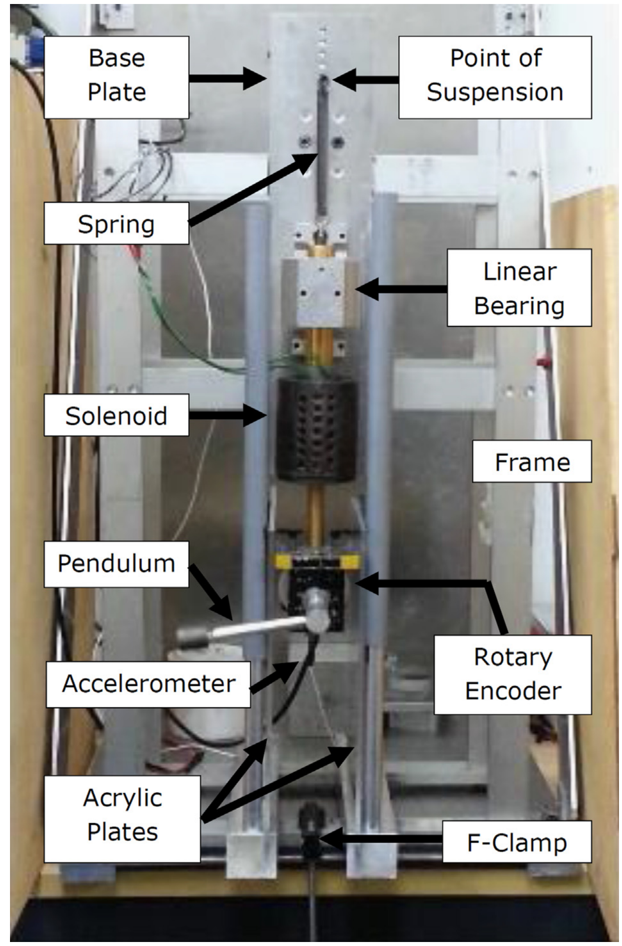


**Fig. 1.** Experimental rig in present work.