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Synchronous rotational motion of parametric pendulums

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ABSTRACT

We study theoretically and experimentally the synchronization phenomenon of two rotating parametric pendulums attached to common elastic support under harmonic excitation. Two types of synchronous states have been identified – complete and phase synchronization. The interactions in the system have been investigated numerically and experimentally. The relation between the synchronization mode and the stability of the rotational motion for a system with flexible support has been studied. It has been demonstrated that the synchronization of pendulums rotating in antiphase is more beneficial from energy harvesting viewpoint than the synchronization in phase. Finally, an influence of the parameter mismatch between the pendulums on their synchronization has been examined.

1. Introduction

The objective of this paper is to study the synchronization of rotational motion in the system of two parametric pendulums subjected to common harmonic excitation. The study has been motivated by a possibility of applying such a system for energy harvesting, as the oscillatory motion can be converted into rotation of pendulums. Consequently, the energy can be harvested from the rotational motion, which is a strongly advantageous alternative to using the energy of oscillations. The challenge of the design of such a structure lies in balancing it properly to guarantee dynamic stability once the pendulum is in motion. Therefore, to compensate for the effect which a single rotating mass exerts on the support, the system consisting of two pendulums is being considered. To achieve the desired balance of forces the pendulums would be required to counter rotate in a synchronized manner. If their responses are synchronized in antiphase the structure remains stable. This section provides an overview of the basics of synchronization theory, reviews the main recent works on the dynamics of parametric pendulum and looks at synchronization in pendulum systems.

1.1. Synchronization theory

The term ‘synchronous’ originates in Greek and denotes something ‘sharing the same time’. The discovery of the synchronization phenomena is directly related to the dynamics of the pendulum. It has been first observed and described in the 17th century by a Dutch researcher, Huygens, on an example of pendulum clocks hanging on the same wall [13]. Recently his experiment has been repeated in Kapitaniak et al. [14]. Huygens observation revealed that the clocks were exactly synchronized, swinging in opposite directions. Even if any disturbance occurred, they were still returning to the synchronized state after some transient time. The reason for this behaviour has been identified in the coupling effect of the beam supporting the clocks, transmitting the vibrations.

Since then synchronization has been detected in various systems and described in many publications. Pikovsky et al. [21] and Blekhman et al. [4] give examples of this phenomenon in mechanical, electrical or biological systems. In the most general sense, occurrence of synchronization between two systems implies existence of some relationship between their responses, without specifying exactly the type of this relation, which can be of a complex nature. Therefore, sometimes synchronization is difficult to detect, as it cannot always be associated with the identity of trajectories. Depending on the relation between the responses, several types of synchronization have been classified. Considering two systems, where $x(t)$ and $y(t)$ denote their trajectories, the following types of synchronization can be distinguished [5]: *Complete synchronization* [CS] is a state at which both phases and amplitudes of the oscillating systems coincide. It can be achieved only in case of identical oscillators when some kind of internal or external coupling between them is introduced. The definition of the CS concept has been introduced by Pecora and Carroll [19] and is said to be a state in which phase trajectories $x(t)$ and $y(t)$ of the coupled systems converge to the same value and

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remain in this relation during the further time evolution. The above concept can be described by the following relation:

$$\lim_{t\to\infty} |x(t)-y(t)| = 0, \tag{1}$$

In practice very often the identity conditions are not fully met. If there is a difference in parameters or noise is present the *imperfect complete synchronization* [ICS] occurs and the synchronizability condition becomes

$$\lim_{t\to\infty} |x(t)-y(t)| < \epsilon, \tag{2}$$

where ϵ is a small parameter. *Phase synchronization* [PS] describes a weaker degree of synchronization. The required coupling between the systems is much lower than in case of CS so that the identity condition is not necessary. It occurs when the phases of oscillations are locked within a certain range. Generally speaking, this correlation does not imply any relation between the amplitudes. The mathematical condition for PS is given by

$$|n\Phi_1(t) - m\Phi_2(t)| < c, \tag{3}$$

where Φ_1 and Φ_2 denote phases of the coupled oscillators n, m integers determining the locking ratio and c is a constant. As a consequence, the frequencies of both systems ω_1 and ω_2 need to be locked as well and satisfy the relation

$$n\omega_1 - m\omega_2 = 0, \tag{4}$$

Based on the type of the system in which synchronization is observed, another classification can be introduced. The first case, based on classical understanding of synchronization, is the synchronization of coupled periodic oscillators. The rhythms of self-sustained periodic oscillators adjust due to their weak interaction, where this adjustment can be described in terms of phase locking and frequency entrainment. The basic model of such coupled system consisting of two oscillators is given by

$$\frac{dx_1}{dt} = f_1(x_1) + \epsilon p_1(x_1, x_2),$$
$$\frac{dx_2}{dt} = f_2(x_2) + \epsilon p_2(x_2, x_1), \tag{5}$$

where ϵ is the coupling parameter. If ϵ vanishes the subsystems become independent and oscillate with their natural frequency. The second type of interaction considered here is the synchronization of periodic oscillators by external force. It can be also observed when a periodic force (or noise) is applied to a group of non-coupled autonomous oscillators. Its occurrence depends not only on the magnitude of forcing but also on the difference between the natural frequency of the system and the forcing one, called detuning parameter. Inside the synchronization region, the system oscillates with the frequency of the external force, while outside quasiperiodic motion can be observed.

Synchronization can also be observed in a noisy system. For such a system the condition for synchronization needs to be modified, for a less rigorous one. The perfect frequency entrainment is not observed any more. A state where frequencies nearly adjust, but still phase slips can be observed, is defined as imperfect phase synchronization (IPS). Finally, synchronization can be observed also for chaotic systems [25,15,6,20,26,23]. Its detection however depends on the type of attractor and can be more complex.

1.2. Parametric pendulum

The parametric pendulum is a system which has been of great interest for years, because of its rich dynamical behaviour [7,3,28,9]. It is a model with numerous engineering applications, including marine structures, superconductor Josephson junction. Many oscillating systems contain pendulum like non-linearity. Therefore, parametric pendulum has been one of the most common systems in the literature illustrating the dynamics of a non-linear oscillator. Among its various responses equilibrium points, oscillations, rotations as well as chaos can be observed.

The physical model of a parametric pendulum and the phase plane representation of the basic responses for unforced undamped system are shown in Fig. 1. The vertical oscillation of the pivot point results in the oscillations or rotations of the pendulum, depending on the initial conditions and forcing parameters. The closed loops marked by 1 and 2 correspond to the oscillations around hanging down position. Once the sufficient amount of energy is supplied the pendulum can escape from the potential well passing the critical case described by separatix (curve 3) and enter rotational motion regime (curves 4).

For many engineering applications, oscillatory responses are of main interest. Rotation of pendulum like systems has been studied before in relation to rotor dynamics and in recent years the research intensified due to potential applications in energy harvesting.

Approximating the escape zone has been the topic of study for Trueba et al. [32], Thompson [31], Bishop and Clifford [9] who used symbolic dynamics approach in their work. Different types of rotations have been classified in [8]. Xu and Wiercigroch [34] derived an analytical solution for rotational motion using multiple scales method where Sofroniou and Bishop [28] applied the harmonic balance method to the problem. Limit of rotational motion existence has been determined analytically by Koch and Leven [16] and Lenci et al. [17], who gave analytical approximation of the rotational solutions including study of their stability.

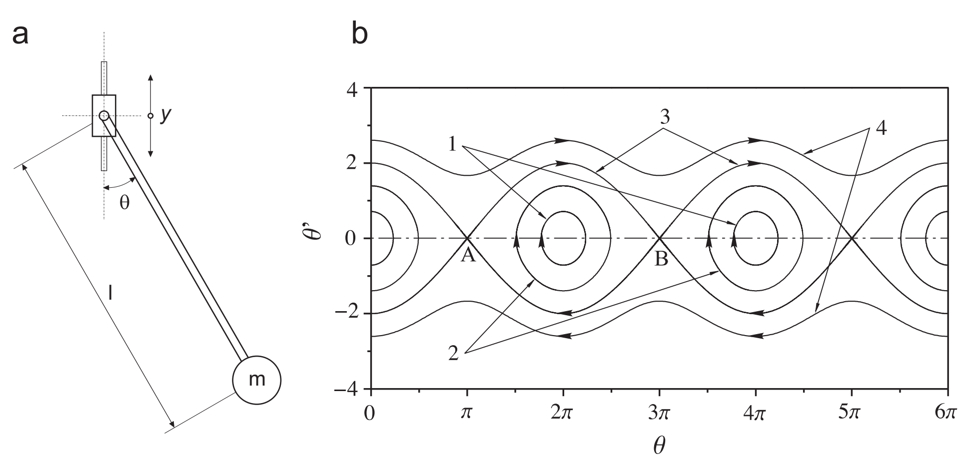

Fig. 1. (a) Physical model of parametrically excited pendulum and (b) phase plane showing different responses of the unperturbed pendulum in terms of pendulum displacement and velocity [35].