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Effects of time-periodic intercoupling strength on burst synchronization of a clustered neuronal network ☆

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ABSTRACT

In this paper, we investigate the effect of time-periodic intercoupling strength on burst synchronization of a clustered neuronal network. We mainly focus on discussing the effects of amplitude and frequency of the time-periodic intercoupling strength on burst synchronization. We found that by tuning the frequency, burst synchrony of the clustered neuronal network could change from higher synchronized states to low synchronized states, and vice versa. While for the amplitude, we surprisingly found that with increasing of the amplitude, burst synchrony of the clustered neuronal network is not always enhanced. We know that synchronization has close relationship with cognitive activities and brain disorders. Thus, our results could give us some useful insight on the important role of time-dependent couplings in neuronal systems.

1. Introduction

Neuronal firing activities are the main means of communication between connected neurons. As one of the two basic neuronal firing activities, bursting occurs when neuronal activity alternates, on a slow time scale, between a quiescent state and fast repetitive spiking. Bursting activities have been observed experimentally in various neuronal systems, such as mammalian thalamic neurons [1], hippocampal CA1 pyramidal cells [2], and sensory neurons [3]. With the aid of bifurcation theory and fast/slow dynamical analysis, bursting activities have been classified into various types mathematically [4]. In sensory information transmission, many results indicate that information encoded by bursts can be transmitted more reliably to cortical networks than information carried by tonic spikes [5]. Moreover, it has been found that some types of epileptic seizures have close relationship with the generation of intrinsic burst firing [6].

Synchronization, as an ubiquitous phenomenon in nature, has also been observed in many nervous systems. Synchronous activity is not only associated with pathological brain states, such as epilepsy, Alzheimer disease and Parkinson; but also linked to various cognitive functions, including expectation, attention, and sensory latencies [7]. Thus, studying synchronization phenomena of neuronal systems is very meaningful for neuroscientists. In the last decade, synchronization phenomena have been extensively studied both theoretically and numerically in neuronal systems [8–11]. However, most of the studies focus on discussing synchronization of spiking neurons. As stated above, bursting activities have important implications in sensory information transmission and even epileptic seizures. Thus, studies on synchronization of bursting neurons are also very important.

Synchronization of bursting neurons includes synchrony on the spiking time scale (spike synchronization), synchrony on the bursting time scale (burst synchronization), and both (complete synchronization) [12,13]. It has been reported recently that burst synchronization is achieved more easily than spike synchronization and complete synchronization [13,14]. And it has been found that burst synchronization of the neuronal systems could be influenced by many factors such as coupling strength and types [15–17], time delays [18] and noise [19,20]. For example, it has been shown that different coupling types could lead to different types of burst synchronization [17], time delays could enhance and induce burst synchronization [18], and noise could be applied to control undesirable synchronized bursting rhythms [19,20]. Additionally, it has been reported that synchronization among chaotically bursting neurons could lead to the onset of regular bursting [21]. Furthermore, burst synchronization transition, a very interesting phenomena, has been reported recently [22,23]. It has been found that,

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coupling strength [22] and distributed time delays [23] can induce burst synchronization transition in a neuronal network, which composed by a group of electrically or chemically coupled bursting neurons.

In recent years, with the development of experimental techniques, it is revealed that the structure of the complex cortical network has hubs [24], small-world [25], hierarchy and clustered characteristics [26,27]. A clustered network consists of a number of groups, where nodes with each group are densely connected, but the linkage among the groups is sparse. The extensive existence of clustering structures in neuronal systems has inspired some researchers to investigate its role in neuronal activities and functions. It is found that hierarchy clustered structure may relate with abilities of the brain to attain criticality and enhanced function [28]. And clustered excitable connections are found to be responsible for the co-exstence of fast spiking variability and slow firing rate fluctuations observed experimentally in neuronal systems [29]. From these works, we can see the significant relationship between clustered network structure and neuronal dynamical activities. Therefore, investigating dynamical behaviors of clustered neuronal network is significant for us to unveiling the underlined mechanisms of complex neuronal activities.

Nowadays, non-linear dynamical studies on clustered neuronal networks have become a hot topic. And some interesting results have been obtained. For example, Yan and Li [30] studied the role of structural connectivity in the origin of the large-scale synchronization of the brain which are thought to be related to epileptic seizures. They found that the global (large-scale) synchronization starts from an increase of local synchronization of some brain areas with high centrality. It implies that the clustered structure plays significant role in the generation of large-scale synchronization. And this work could give some indications on epilepsy control. Moreover, Li and Zhou [31] found that clustered structure could help the brain to attain criticality and enhanced function. For anti-phase synchronization widely observed in cortical neural network, it is found that except for time delay, other structural factors in cortical neuronal network such as modular organization and the coupling types could also play important roles. Meanwhile, it is also discovered that both inter- and intracoupling strength could induce burst synchronization in a clustered neuronal network with each cluster being regular [32]. We know that synchronization not always plays a positive role in neuronal system. At some situations, we need to suppress it. Lameu and Batista et al. [33] investigated how to suppress burst synchronization in clustered scale-free neuronal network. They found that bursting synchronization could be suppressed by deactivating a single neuron which is highly connected in the network.

In the above studies, the coupling strength are usually considered to be constants. However, as we know, interacting strength between neurons usually changes with time because of the neuronal characteristics of synaptic plasticity. In this paper, we discuss burst synchronization in a clustered neuronal networks with a time-periodic intercoupling strength. And we focus on discussing what the influences of this time-changing coupling strength on the burst synchronization in the studied clustered neuronal network, which consists of electrically coupled bursting Hindmarsh–Rose (HR) neurons. Here, burst synchronization (or synchronization of burst) means that the bursting of each neuron inside the network starts and ends nearly simultaneously, while spikes inside them are not synchronized. In the followings, we introduce the mathematical models which could describe the clustered neuronal network and the measurement to quantify the burst synchronization degree at first. And then we exhibit the obtained simulation results about the effect of time-periodic intercoupling strength (TPICS) on burst synchronization. After that, we give some discussions about the effect of noise on the obtained results and what happens if the subnetworks have different number of neurons. Finally, we draw a conclusion from this work.

2. Model and measurements

In this paper, Hindmarsh–Rose (HR) [34] neuronal model is applied as the basic building block, and neurons are connected with each other with electrical synapses. The studied neuronal network is described by the following equations:

$$\begin{aligned}\dot{x}_{I,i} &= y_{I,i} - a x_{I,i}^3 + b x_{I,i}^2 - z_{I,i} + I_{ext} + \epsilon_{intra} \\ &\quad \times \sum_j A_I(i,j)(x_{I,j} - x_{I,i}) + \epsilon_{inter} \sum_J \sum_j B_{I,J}(i,j)(x_{J,j} - x_{I,i}), \\ \dot{y}_{I,i} &= c - d x_{I,i}^2 - y_{I,i}, \\ \dot{z}_{I,i} &= r[s(x_{I,i} - x_0) - z_{I,i}], \end{aligned} \tag{1}$$

where x represents the membrane action potential, y represents the fast current, like that of Na^+ or K^+, and z associates with the slow current, for example, a current of Ca^{2+}. The parameters are taken as $a=1.0$, $b=3.0$, $c=1.0$, $d=5.0$, $r=0.006$, $s=4.0$, $x_0=-1.6$, and external current $I_{ext}=3.0125$, such that an isolated neuron produces chaotic bursting activity. The subscript pairs (I,i) represent the i-th neuron in the I-th cluster. The matrix $A_I=(A_I(i,j))$ is a connectivity matrix for neurons inside the I-th cluster: $A_I(i,j)=1$ if neuron i is connected to neuron j inside the I-th cluster, $A_I(i,j)=0$ otherwise, and $A_I(i,i)=0$. The matrix $B_{I,J}=B_{I,J}(i,j)$ is also a connectivity matrix, which represents the connections between neurons which belong to different clusters: $B_{I,J}(i,j)=1$ if the i-th neuron in the I-th cluster is connected to j-th neuron in the J-th cluster, $B_{I,J}(i,j)=0$ otherwise. ϵ_{intra} is the coupling strength for neurons inside the cluster, while ϵ_{inter} is the coupling strength for neurons between different clusters. If ϵ_{intra} is large, it will play a dominant role, which is disadvantageous for us to investigate the effect of intercoupling strength. While if ϵ_{intra} is small, the intercoupling strength should be large enough to observe synchronization phenomena. Therefore, in order to discuss the effects of time-periodic intercoupling strength on the above neuronal networks efficiently, ϵ_{intra} takes an intermediate value 0.0052. Actually, we can also set it as other intermediate values without changing the results. And the intercoupling strength ϵ_{inter} is taken as a time-periodic coupling term, formulated as

$$\epsilon_{inter} = \epsilon_0(1+\cos(\omega t)), \tag{2}$$

where ϵ_0 and ω are the amplitude and frequency of the intercoupling time-periodic strength.

The clustered neuronal network considered here consists of N neurons, which are grouped into M clusters, where each cluster contains $n=N/M$ neurons [39,40]. In each cluster, the neurons are arranged on a ring so that the subnetwork is regular. Each neuron connects to its $2k$ nearest neighbors. Namely, each cluster has the same intracoupling matrix A_I. While, we assume that links between neurons from different clusters exist with probability p, i.e., $B_{I,J}(i,j)$ takes value of 1 with probability p and 0 with probability $1-p$. Furthermore, for simplicity, we just consider bidirectional coupling cases.

Moreover, in order to quantify the degree of burst synchronization, we introduce the order parameter which is calculated as [14]

$$R = \frac{1}{N}\left|\sum_{j=1}^{N} \exp[i\phi_j(t)]\right|, \tag{3}$$

where $\phi_j(t)$ is the burst phase for the j-th neuron at the time t, and can be presented as

$$\phi_j(t) = 2\pi\frac{t-T_{j,k}}{T_{j,k+1}-T_{j,k}}, \quad T_{j,k} \le t \le T_{j,k+1}, \tag{4}$$

where $T_{j,k}$ is the moment at which the k-th burst of the j-th neuron starts, $j=1,\ldots,N$. To identify the burst starting times, spike trains of each neuron are filtered to quasi-square waves at first. And then