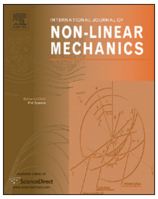
International Journal of Non-Linear Mechanics 70 (2015) 126–133

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm



Stress-free layers in photoinduced deformations of photoelastomer beams

Zoltán Ábrahám [a], György Károlyi [b,*]

[a] Faculty of Civil Engineering, Budapest University of Technology and Economics, Műegyetem rkp. 3., H-1111 Budapest, Hungary
[b] Institute of Nuclear Techniques, Budapest University of Technology and Economics, Műegyetem rkp. 9., H-1111 Budapest, Hungary

ARTICLE INFO

Article history:
Received 4 December 2013
Received in revised form
10 August 2014
Accepted 8 October 2014
Available online 19 October 2014

Keywords:
Photoelastomer beam
Nematic liquid crystal
Bending
Non-linear stress distribution

ABSTRACT

Nematic liquid crystals combined with long molecular chains to form liquid crystal elastomers are capable of large extension. When such liquid crystal elastomers contain azo dyes to constitute photoelastomers, illumination can trigger large contraction. Beams made from such photoelastomers possess a non-uniform illumination and hence photostrain across their cross-section, resulting in bending and highly non-linear stress distribution. Due to the non-linear stress distribution, there can be more than one stress-free layers within the beam. In this paper, we present a dimensionless parametric study of nematic photoelastomer beams under the combined effects of light and mechanical loads. We show how the number of stress-free layers depends on three dimensionless parameters. The paths traced out by the system in the space of dimensionless parameters by varying the different real parameters are investigated, showing how the number of stress-free layers changes when e.g. the thickness or the mechanical load of the elastomer beam is varied. These results are important if the strain induced director rotation is not negligible.

1. Introduction

Stiff, rod-like molecules constituting nematic liquid crystals align below the nematic–isotropic phase transition temperature, resulting in uniaxial orientational order [1]. When rubber is formed from nematic liquid crystals by including a network of long molecular chains, the resulting liquid crystal elastomer becomes capable of large extension (up to 400%) when taken through its nematic–isotropic phase transition temperature [1–3]. When the liquid crystal elastomer contains azo dyes (e.g. azobenzene) or other photoisomerizable molecular rods, under the effect of light, photon absorption implies a *trans* → *cis* transition and the shape of these rods becomes strongly kinked [4,5], see Fig. 1a. This effect dilutes the nematic order and results in the contraction of the liquid crystal elastomer. This contraction of photoelastomers is comparable in magnitude to that observed for thermal nematic–isotropic phase transition [3,6].

If a beam built from a photoelastomer is illuminated from one side, as in Fig. 1b, light penetrates the beam. However, due to absorption, the distribution of light across the depth of the beam is not uniform, resulting in an uneven photostrain across the beam. This inhomogeneity implies the bending of the beam [7–11].

In the simplest approximation, according to Beer's law, photon absorption causes light intensity to decay exponentially with the depth of penetration [7]. Since light intensity governs the nematic order, the latter is also not uniform across the thickness of the elastomer beam. As a consequence, a non-uniform photoinduced strain in nematic photoelastomer beams develops, decaying also exponentially with the depth. The induced stress will hence also be non-linear across the cross-section of the photoelastomer beam, and up to two stress-free layers can develop [7,8] when the beam is illuminated from one side as in Fig. 1b.

In the more general case, when light intensity is not assumed to decay exponentially with depth, one has to take into account how the *trans* and *cis* fractions of the incorporated azo dye vary with depth as a consequence of absorption [8–11]. In equilibrium, the *trans* → *cis* transition caused by illumination is balanced by the *cis* → *trans* back reaction in each layer of the beam. The light intensity at a certain layer is governed by the absorption in the layers towards the illuminated side of the beam. This non-linear effect causes an even more complex stress distribution across the beam than in the case when Beer's law is valid. As a consequence, up to three stress-free layers can be found across the cross-sections of such beams [8,10,11].

In this paper, we offer a systematic dimensionless parametric study of nematic photoelastomer beams under the effects of incoming light and imposed mechanical load (eccentric force). We show analytically

---

[*] Corresponding author.

http://dx.doi.org/10.1016/j.ijnonlinmec.2014.10.011
0020-7462/

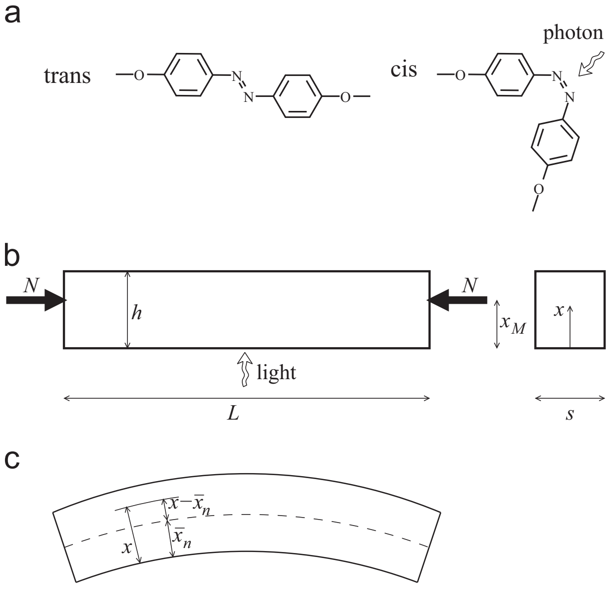


Fig. 1. (a) The *trans* → *cis* transformation of azobenzene under photon absorption, adapted from Ref. [5]. (b) Layout of the illuminated beam under eccentric load N. (c) Deformed shape with neutral layer at $\bar{x}_n$.

how the number of stress-free layers depends on the dimensionless parameters of the problem. It turns out that the number of stress-free layers depends on three parameters: beside the intensity of the incoming light, two more dimensionless parameters determine the behaviour, both of them being complicated functions of the real parameters of the beam, including the mechanical load, the elastic properties, the geometrical parameters and material properties. We also show the paths traced out by the system in the space of dimensionless parameters when the mechanical loads or the characteristic decay length of the illumination are varied. This sheds light on how the number of stress-free layers depends on the parameters. This can be important because there is evidence that the orientational order can depend on whether parts of the beam are in tension or in compression [12]. Hence a more refined theory of bending of photoelastomer beams under incident light should incorporate in a self-consistent way which layers are in tension or in compression, being separated by the stress-free layers investigated in this paper.

2. Illuminated photoelastomer beam

We follow Ref. [8] to derive the strain induced by light and the corresponding strain and stress distribution across the photoelastomer beam. Then we formulate the dimensionless equation governing the number of stress-free layers.

2.1. Light induced strain

When azobenzene absorbs light, it undergoes the *trans* → *cis* transition. Hence the *trans* fraction n_t within the sample decays, but at the same time it is increased by the *cis* → *trans* spontaneous backreactions:

$$\dot{n}_t = -\Gamma I n_t + n_c/\tau, \tag{1}$$

where $n_c = 1 - n_t$ is the *cis* fraction, Γ is the constant rate of transition assumed to be independent of nematic order, τ is the average lifetime of *cis* state that gives constant rate of thermal back relaxation, and dot indicates time derivative. The light intensity is denoted by I. Here we ignored [8] the photo-induced *cis* → *trans* back-reaction [17] and also the effects of temperature change due to illumination. We look for the steady state $\dot{n}_t = 0$, hence we find

$$n_c(x) = \frac{I(x)}{I_c + I(x)}, \quad n_t(x) = \frac{I_c}{I_c + I(x)}, \tag{2}$$

with $I_c = 1/\Gamma\tau$. In steady state the *cis* and *trans* fractions and light intensity depend only on depth x measured from the illuminated surface of the beam, see Fig. 1b. Light decays within the material due to absorption as $\mathrm{d}I/\mathrm{d}x = -n_t I/d$, where d is the attenuation length, another material constant. Absorption due to non-dye matrix is ignored for simplicity, it would only rescale the value of $I(x)$ [9]. Substituting (2) implies

$$\frac{\mathrm{d}I(x)}{\mathrm{d}x} = -\frac{1}{d}\frac{I(x) I_c}{I(x) + I_c}. \tag{3}$$

Integrating this equation with $I(0) = I_0$ (light intensity at the illuminated surface) gives

$$\log\left(\frac{I(x)}{I_0}\right) + \frac{I(x) - I_0}{I_c} = -\frac{x}{d}. \tag{4}$$

In the case when I_c is large ($I_c \gg I_0$), that is, $\Gamma\tau I_0 \ll 1$, we are in Beer's law regime, when the *cis* fraction is small, *trans* fraction is close to one, which is the case when illumination is low. In this special case we find $\log(I(x)/I_0) = -x/d$, i.e., an exponential decay $I(x) = I_0 e^{-x/d}$ of light intensity with depth.

The strain $\varepsilon_r(x)$ imposed by illumination is assumed to be proportional to the *cis* fraction $n_c(x)$, valid for low azoconcentration (up to 10 wt%) [11]:

$$\varepsilon_r(x) = -A n_c(x), \tag{5}$$

where $A > 0$ is a material constant, assumed not to depend on the nematic order. The negative sign is due to the fact that the *trans* → *cis* transition implies a strong kink in the azobenzene containing molecules disrupting the nematic order, hence resulting in contraction, see Fig. 1a. Using Eqs. (5), (2) and (3) we find

$$\varepsilon_r(x) = -\frac{A I(x)}{I(x) + I_c} = \frac{A d}{I_c}\frac{\mathrm{d}I(x)}{\mathrm{d}x}. \tag{6}$$

In the large I_c limit (Beer's law), this simplifies to

$$\varepsilon_r(x) = -\frac{A}{I_c} I(x). \tag{7}$$

This implies that Beer's law is valid in the $I_c \to \infty$, $A \to \infty$ limit as long as A/I_c remains finite.

2.2. Light induced bending

As the photoinduced strain $\varepsilon_r(x)$ depends on depth x, the non-uniform and non-linear photostrain across the cross-section of the beam leads to bending. Assuming that the cross-sections remain planar, a non-linear stress distribution $\sigma(x)$ develops across the cross-section implying an elastic strain

$$\varepsilon_b(x) = \sigma(x)/E, \tag{8}$$

where E is Young's modulus of linear elasticity, which is assumed not to depend on the nematic order. The total strain is hence

$$\varepsilon(x) = \varepsilon_b(x) + \varepsilon_r(x). \tag{9}$$

With a stress-free surface at $x = x_n$, i.e., $\sigma(x_n) = 0$, we find $\varepsilon_b(x_n) = 0$ and hence $\varepsilon(x_n) = \varepsilon_r(x_n)$. Similarly, with a strain-free neutral surface at $\bar{x}_n$, i.e., $\varepsilon(\bar{x}_n) = 0$, we can assume that the cross-sections rotate around the $x = \bar{x}_n$ axis with a curvature κ, and hence

$$\varepsilon(x) = \kappa(x - \bar{x}_n). \tag{10}$$

We can express $\kappa\bar{x}_n$ from (10), and also from (10) after substituting x_n in place of x. Then equating these two we find

$$\varepsilon_b(x) + \varepsilon_r(x) = \varepsilon(x) = \varepsilon(x_n) + \kappa(x - x_n) = \varepsilon_r(x_n) + \kappa(x - x_n). \tag{11}$$