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Non-linear analysis and quench control of chatter in plunge grinding

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ABSTRACT

This paper aims at mitigating regenerative chatter in plunge grinding. To begin with, a dynamic model is proposed to investigate grinding dynamics, where eigenvalue and bifurcation analyses are adopted, respectively, for prediction of grinding stability and chatter. Generally, it is found that most grinding chatter is incurred by subcritical Hopf bifurcation. Compared with supercritical instability, the subcritical generates coexistence of stable and unstable grinding in the stable region and increases chatter amplitude in the chatter region. To avoid these adverse effects of the subcritical instability, bifurcation control is employed, where the cubic non-linearity of the relative velocity between grinding wheel and workpiece is used as feedback. With the increase of feedback gain, the subcritical instability is transformed to be supercritical not only locally but also globally. Finally, the conditionally stable region is completely removed and the chatter amplitude is decreased. After that, to further reduce the chatter amplitude, quench control is used as well. More specifically, an external sinusoid excitation is applied on the wheel to quench the existing grinding chatter, replacing the large-amplitude chatter by a small-amplitude forced vibration. Through the method of multiple scales, the condition for quenching the chatter is obtained.

1. Introduction

Machining vibration impoverishes surface finish and breaks cutting tools [1]. Such undesired instability can be self-excited or externally forced [2]. Generally, most machining vibrations are self-excited, called chatter and induced by interactions between parts and cutting tools [3]. The machining chatter can be classified into different types, including the frictional, the regenerative, the mode-coupling and the thermomechanical chatter [4]. Among these chatters, this paper concentrates on the most common form, the regenerative chatter.

Briefly speaking, the regenerative chatter is induced by regeneration of workpiece surface. In the machining, small waves in the workpiece surface are regenerated by pass of the cutting tool [1]. Subsequently, the surface regeneration causes variation of chip thickness, thus the fluctuating cutting force induces chatter. This mechanism was first revealed by Arnold [5], who reported turning chatter. Then the regenerative machine tool chatter theory was first presented by Tobias [6]. After that, the chatter in the turning and the milling are investigated thoroughly by many researchers [7–14], establishing the regenerative theory. Compared with the turning or the milling, the grinding regeneration exists in both workpiece and grinding wheel surfaces [15], called a doubly regenerative problem [16]. The feature of the grinding was captured by Thompson, who used a kinematic model to discuss the stability of plunge grinding [17–20]. Afterwards, Li and Shin [21] improved this model and obtained more precise results, by using numerical simulations. Later on, similar kinematic mode and simulation method are employed by Weck [22], who investigated the chatter in transverse grinding.

Except these numerical works, dynamical models were presented for stability analysis and chatter prediction of the grinding as well. Yuan et al. [23] proposed a model with double time delay effects to study the grinding dynamics, where the delays represent the regeneration in the workpiece and wheel surfaces respectively. In their work, eigenvalue and perturbation analyses were used to study the grinding stability and the chatter vibration. Later on, this model is linearized by Liu and Payre [16] to investigate the effects of dominating parameters on the grinding stability. Taking non-linear effects of regenerative grinding force into consideration, Chung and Liu [24] extended this model for the chatter prediction.

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Using perturbation incremental scheme (PIS) [25], they observed the chatter motions induced by supercritical Hopf bifurcation. After that, Yan et al. [26] used this model to study non-linear grinding chatter, which found Bautin bifurcation in the plunge grinding. Afterwards, Kim et al. [27] obtained similar results in the analysis of the chatter in the transverse grinding.

In the analyses of chatter, it is found that most chatter is triggered off by subcritical Hopf bifurcation [26–30]. Compared to the supercritical, the subcritical is much more troublesome [31]. This type of instability not only causes large-amplitude vibration but also erodes the stable regions, making it conditionally stable [26]. In the region of conditional stability, the coexistence of stable and unstable cutting shrinks available parameter region, which is always undesired in the machining. To eliminate the subcritical instability in the turning, Nayfeh [1] and Pratt and Nayfeh [31] proposed bifurcation control, where non-linear state feedback was used to transform the bifurcation pattern. In addition, another active control method, called quench control, was adopted by Nayfeh to further suppress the turning chatter [32,1]. However, in the respect of the grinding chatter, the effects of the bifurcation and the quench control have not been discussed up to now.

To eliminate the undesired non-linear effect in the grinding, the bifurcation and the quench control are employed in this paper. Firstly, the dynamic model of the grinding process is given in Section 2. After that, Section 3 studies the grinding dynamics by eigenvalue analysis [16], perturbation analysis [33], DDEBIFTOOL [34] and numerical simulation [23]. It is seen that the chatter mostly occurs through the subcritical Hopf bifurcation; therefore, the bifurcation control is used in Section 4. With the increase of the feedback gain, the bifurcation is transformed not only locally but also globally. As a consequence, the conditionally stable region is removed and the chatter amplitude is reduced. Thereafter, to further mitigate the chatter, the quench control is adopted in Section 5. More specifically, a sinusoid external excitation is applied on the grinding wheel. Using the method of multiple scales, we find the condition to successfully quench the grinding chatter. With the increase of the force amplitude, the large-amplitude chatter is replaced by a small-amplitude forced vibration; thus the machining tolerance is maintained [31].

2. Dynamical model of plunge grinding

A grinding process uses a rotating grinding wheel to grind workpiece surface, removing redundant materials to achieve machining precision. In the grinding, the cylindrical workpiece rotates while the grinding wheel is plunged into the workpiece to feed the cutting tool with new materials. For the grinding, the cutting force is generated by the wheel–workpiece interaction. Constant interaction creates constant grinding force, while fluctuating cutting depth generates time-varying force. Subjected to the regenerative force, the workpiece and the wheel are excited to vibrate. When the system is not well damped, grinding chatter occurs.

The grinding dynamics is depicted in Fig. 1. As seen, the workpiece is a cylindrical beam, with its ends supported by two tailstocks. The beam is of mass density ρ (kg m^{-3}), Young's modulus E (N m^{-2}), damping c_w (N s m^{-2}), length L (m), and radius r_w (m). The grinding wheel is regarded as a damped spring–mass system, with mass m_g (kg), stiffness k_g (N m^{-1}), damping c_g (N s m^{-1}), and radius r_g (m). In Fig. 1, the grinding force F_g (N) is generated at $S=P$, where the wheel grinds the workpiece with a rotary speed of N_g (rpm). To feed the grinder, the workpiece rotates with a speed of N_w (rpm), and the wheel is plunged into the workpiece with a feed speed f (m rev^{-1}).

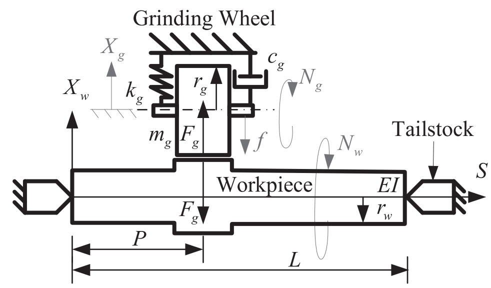

Fig. 1. Schematic of the plunge grinding process.

2.1. Dynamic model

To describe the grinding dynamics, the wheel and the workpiece are assumed to be a spring–mass system and a Euler–Bernoulli beam respectively. Correspondingly, the workpiece $X_w(t,S)$ (m) and wheel $X_g(t)$ (m) displacements are governed by

$$m_g\frac{d^2X_g(t)}{dt^2}+c_g\frac{dX_g(t)}{dt}+k_gX_g(t)=F_g,$$
$$\rho A\frac{\partial^2X_w(t,S)}{\partial t^2}+c_w\frac{\partial X_w(t,S)}{\partial t}+EI\frac{\partial^4X_w(t,S)}{\partial S^4}=-\delta(S-P)F_g, \tag{1}$$

where $A=\pi r_w^2$ (m^2) is the cross-sectional area of the workpiece, $I=\pi r_w^4/4$ (m^4) is the inertia moment of the workpiece, $\delta(S-P)$ (m^{-1}) is the Dirac delta function, and P (m) is the position of the wheel. As the workpiece is supported by the tailstocks, its boundary conditions are given by [35]

$$X_w(t,0)=\frac{\partial^2X_w(t,0)}{\partial S^2}=0,\quad X_w(t,L)=\frac{\partial^2X_w(t,L)}{\partial S^2}=0. \tag{2}$$

In Eq. (1), the grinding force F_g is a non-linear function of grinding depth D_g (m). Based on the model proposed by Li et al. [36], the force is presented as

$$F_g=\begin{cases}WK\left(\dfrac{r_wN_w}{r_gN_g}\right)D_g+WC\left(\dfrac{r_wN_w}{r_gN_g}\right)^{\mu}D_g^{(1+\mu)/2} & \text{if } D_g>0,\\ 0 & \text{if } D_g\le 0,\end{cases} \tag{3}$$

where W (m) is the cutting width, K (N m^{-2}) is the cutting stiffness, C (N m$^{-(3+\mu)/2}$) is the specific friction force, and $\mu\in(0,\frac{2}{3})$ an exponential coefficients. In Eq. (3), $WK(r_wN_w/r_gN_g)D_g$ is the chip formation component of the grinding force, representing grains cutting the workpiece materials. By contrast, $WC(r_wN_w/r_gN_g)^{\mu}D_g^{(1+\mu)/2}$ is the friction force, standing for the sliding friction between the tip flats of active grains and the workpiece [36]. Moreover, Eq. (3) involves the situation of losing contact as well. When negative grinding depth turns up ($D_g<0$), the grinding force disappears ($F_g=0$).

Based on the doubly regenerative theory, the grinding depth D_g depends on the feed f as well as the relative displacement between the wheel and the workpiece $\Delta(t)=X_g(t)-X_w(t,P)$. For each revolution of the workpiece, the wheel is plunged into the workpiece with the feed f. Meanwhile, the wheel and the workpiece are subjected to the grinding force F_g, backing off with the displacements, $X_g(t)$ and $X_w(t,S)$. As seen in Fig. 2, the current grinding depth involves the workpiece regeneration $\Delta(t)-\Delta(t-T_w)$ [16,24], where $T_w=60/N_w$ is the rotational period of the workpiece. In addition, in consideration of the wheel wear $g\Delta(t-T_g)$, the grinding depth is given by

$$D_g=f-\Delta(t)+\Delta(t-T_w)+g\Delta(t-T_g) \tag{4}$$

where $T_g=60/N_g$, and $g\in(1,\frac{1}{5000})$ is the grinding ratio, representing the wear speed of the wheel [37]. A soft grinder ($g=1$) is prone to wear, while a hard one ($g=\frac{1}{5000}$) is wear-resistant.