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Multi-physics dynamics of a mechanical oscillator coupled to an electro-magnetic circuit

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ABSTRACT

The dynamics of a non-linear electro-magneto-mechanical coupled system is addressed. The non-linear behavior arises from the involved coupling quadratic non-linearities and it is explored by relying on both analytical and numerical tools. When the linear frequency of the circuit is larger than that of the mechanical oscillator, the dynamics exhibits slow and fast time scales. Therefore the mechanical oscillator forced (actuated) via harmonic voltage excitation of the electric circuit is analyzed; when the forcing frequency is close to that of the mechanical oscillator, the long term damped dynamics evolves in a purely slow timescale with no interaction with the fast time scale. We show this by assuming the existence of a slow invariant manifold (SIM), computing it analytically, and verifying its existence via numerical experiments on both full- and reduced-order systems. In specific regions of the space of forcing parameters, the SIM is a complicated geometric object as it undergoes folding giving rise to hysteresis mechanisms which create a pronounced non-linear resonance phenomenon. Eventually, the roles played by the electro-magnetic and mechanical components in the resulting complex response, encompassing bifurcations as well as possible transitions from regular to chaotic motion, are highlighted by means of Poincaré sections.

1. Introduction

This paper addresses the dynamics of a system in which a mechanical linear oscillator is non-linearly coupled to a linear electric circuit through an electromagnet. The interest on this type of systems stems from their growing use in mechatronics for a variety of applications such as signal processing, sensing and actuation [1]. Depending on the field of application, mechanical vibratory devices integrated with electronics are made of components ranging in size from nano to micro and macro dimensional scales. Examples of such systems are abundant in modern engineering design, ranging from electric propulsion of land vehicles and ships to micro electromechanical and biomedical devices [2,3]. Regardless of their scale they represent an interesting class of dynamical systems as they involve multi-physics dynamics encompassing coupling among the mechanical, electrostatic, electromagnetic and fluid fields [4–6]. Lumped- or distributed-parameters analytical models, finite-element models and reduced-order models have been proposed for modeling and simulate these multi-field dynamical systems. Moreover, as it is well known, these devices are prone to exhibit non-linear behavior arising either from their inherent mechanical nature and/or from the coupling itself. For small scale devices (M/NEMS), the mechanical subsystem design includes beams, plates or lumped masses that are often characterized by low damping, high flexibility and the presence of non-linear potential fields; thus, the resulting dynamics is strongly influenced by unavoidable non-linear behaviors that can be even actively exploited. An exhaustive overview of research activity on non-linear behaviors arising in micro- nano/resonators can be found in [7]. In this review a number of studies addressing directly and parametrically excited resonators as well as arrays of coupled M/NEMS exhibiting synchronization and vibration localization are reported. A more recent comprehensive overview of MEMS dynamics can be found in [8], in which the sources of non-linearities arising in MEMS modeling due to forcing, damping and stiffness are discussed.

As far as the coupling between electrical and mechanical subsystems, the non-linear nature of the electro-magnetic coupling force has to be taken into account. In this respect, an asymptotic approach for self-excited oscillations was discussed in [9] for a system modeled through two coupled second order differential equations. More recently, in [10], the linear and weakly non-linear dynamics of a coupled electro-mechanical system was studied. In this paper the mechanical system was modeled by a standard second-order differential equation with respect to displacement whereas the electro-magnetic one, after neglecting

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the capacitance term, was governed by a first-order differential equation with respect to current. The near-resonant vibrations resulting from forcing the mechanical oscillator were analyzed by means of a classical perturbation method. Softening behavior and jump phenomena were thereby parametrically investigated with respect to the electro-mechanical and the electro-magnetic coupling parameters.

The system analyzed in this paper is modeled following [9] and the dynamics resulting from exciting the mechanical oscillator via harmonic voltage applied to the electric circuit is considered. The global dynamics of such multi-physics system can be effectively handled to make it functioning either as a sensor or an actuator for applications in the micro electromechanical context. After having introduced the governing differential problem in Section 2, the slow–fast time decomposition analysis is addressed in Section 3. Next, in Section 4, the interaction between the slow and fast dynamics is numerically explored leading, in Section 5, to the analytic computation of the Slow Invariant Manifold (SIM). The main features of the slow dynamics of the full order system (FOS) are described in Section 6. In the following section the approximations to the SIM are described up to the tenth order reduced slow system and the comparison between the various order reduced systems and the full one is carried out by means of bifurcation diagrams and frequency–amplitude plots. The complex chaotic interaction dynamics is eventually addressed in Section 8 through Poincaré sections and concluding remarks emerging from the analysis are reported.

2. A multi-physics coupled oscillators system

In Fig. 1 a dynamical system consisting of coupled mechanical and electro-magnetic subsystems is sketched. The mechanical part is a linear oscillator coupled non-linearly through an electromagnet to a linear electric circuit. Let m, c, k denote respectively the mass, dissipation, spring parameters of the linear oscillator. Let L, C, R denote respectively the inductance, capacitance, and resistance of the electrical oscillator. The variables x and q denote the mechanical and electrical displacement (charge), respectively.

The velocity of the mass and the electric current are $v \equiv \dot{x}, i \equiv \dot{q}$, respectively. The variables f and e denote external mechanical forcing and voltage excitation, respectively. This multi-physics coupled system is a non-linear dynamical system with its coupling non-linearity stemming from the dependence of the inductance on the displacement and possibly velocity of the metallic oscillator mass. In general, as known from electro-magnetic theory, the inductance is a function of the displacement of the metallic mass. In this work we

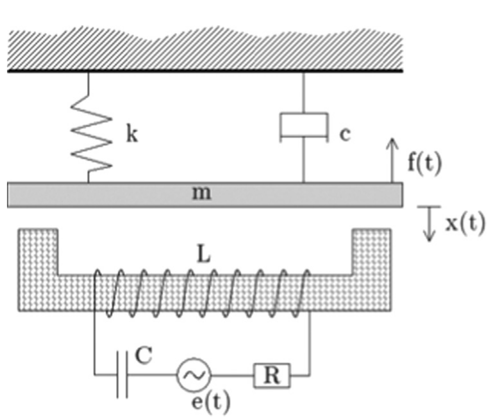

Fig. 1. Mechanical system coupled via a magnetic field to a magneto-electrical system.

assume that the magnet is positioned at a characteristic distance l_0 from the equilibrium position of the oscillating metallic mass. We assume that the inductance is approximated by a linear function of the metallic mass displacement [9]:

$$L(x) = L_0 + L_1 x \tag{1}$$

in which $0 < x < l_0$. The linear natural frequencies and linear dissipation factors of the mechanical and electrical oscillators are given, respectively, by

$$\omega_m^2 \equiv \frac{k}{m}, \quad \omega_e^2 \equiv \frac{1}{L_0 C}, \quad \zeta_m \equiv \frac{c}{2\sqrt{km}}, \quad \zeta_e \equiv \frac{R\sqrt{C}}{2\sqrt{L_0}} \tag{2}$$

The electro-mechanical inertia and non-linear inductance coupling parameters are defined as

$$\epsilon \equiv \frac{L_0}{2m}, \quad \alpha \equiv \frac{L_1}{L_0} \tag{3}$$

We introduce the external force-per-mass and voltage-per-inductance excitations:

$$\hat{f} \equiv \frac{f}{m}, \quad \hat{e} \equiv \frac{e}{L_0} \tag{4}$$

The equations of motion of the coupled electro-magneto-mechanical system are given by the following two coupled second order ordinary differential equations:

$$\begin{aligned} &\ddot{x} + 2\zeta_m \omega_m \dot{x} + \omega_m^2 x = \epsilon\alpha \dot{q}^2 + \hat{f} \\ &(1+\alpha x)\ddot{q} + (2\zeta_e \omega_e + \alpha \dot{x})\dot{q} + \omega_e^2 q = \hat{e} \end{aligned} \tag{5}$$

These equations are similar to those in reference [9] where self-excited oscillation of an analogous system is addressed. The dynamics of the system excited by harmonic voltage while the external forcing in the mechanical part is absent, i.e. $\hat{f} = 0$, is studied. The interest lies in investigating how the mechanical part will respond when it is actuated by the electrical part; the question being to ascertain whether the steady dynamics of the former is determined by the dynamics of the latter. The study is carried out by combining direct numerical simulations and geometric mechanics formulations in the form of invariant manifolds of the system response. Invariant manifolds furnish global picture of the dynamics involving the time scales of the mechanical and the magneto-electrical subsystems. The slow–fast decomposition approach presents a global geometric picture of the dynamics and it allows anticipating bifurcations due to interactions between the oscillators.

3. Slow–fast time decomposition analysis

The theory of Geometric Singular Perturbations (GSP) [11] provides the proper mathematical framework to support geometrically a slow–fast decomposition of the coupled multi-physics dynamics. Following [12,13], we define the ratio of the linear natural frequencies, $\mu \equiv \omega_m / \omega_e$, as the natural singular parameter of the coupled system. In the sequel, after scaling the time, $\tau \equiv \omega_m t$, and using the following state or phase space transformation:

$$x_1 = x, \quad x_2 = x', \quad x_3 = \tau, \quad z_1 = \frac{q}{\mu^2}, \quad z_2 = \frac{q'}{\mu} \tag{6}$$

the second order system (5) assumes the autonomous first order description through the following state space formulation:

$$\begin{aligned} &x_1{}' = x_2 \\ &x_2{}' = -x_1 - 2\zeta_m x_2 + \epsilon\alpha\mu^2 z_2^2 \\ &x_3{}' = 1 \\ &\mu z_1{}' = z_2 \\ &\mu(1+\alpha x_1) z_2{}' = -z_1 - 2\zeta_e z_2 - \alpha\mu x_2 z_2 + \hat{E}(x_3) \end{aligned} \tag{7}$$