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A study on fatigue crack growth in the high cycle domain assuming sinusoidal thermal loading

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ABSTRACT

The assessment of fatigue crack growth due to turbulent mixing of hot and cold coolants presents significant challenges, in particular to determine the thermal loading spectrum and the associated crack growth. The sinusoidal method is a simplified approach for addressing this problem, in which the entire spectrum is replaced by a sine-wave variation of the temperature at the inner pipe surface. The loading frequency is taken as that which gives the shortest crack initiation and growth life. Such estimates are intended to be conservative but not un-realistic. Several practical issues which arise with this approach have been studied using newly-developed analytical solutions for the temperature and stress fields in hollow cylinders, in particular the assumptions made concerning the crack orientation, dimensions and aspect ratio. The application of the proposed method is illustrated for the pipe geometry and loadings conditions reported for the Civaux 1 case where through wall thermal fatigue cracks developed in a short time, but the problem is relevant also for fast reactor components.

1. Introduction

Quantifying the thermal fatigue damage and the subsequent crack growth which may arise due to associated thermal stresses from turbulent mixing or vortices in light water reactor (LWR) piping systems and fast reactor components remains a demanding task and much effort continues to be devoted to experimental studies and development of models with different levels of complexity [1–5]. The problem, sometimes referred to as thermal striping, is characterised by rapid fluid temperature fluctuations with random frequencies and the resulting through-wall stress distributions in the pipe are strongly nonlinear. Due to the bi-axial stress state and the strong stress gradients, cracks initiate at multiple locations on the inside surface of the pipe where they form networks. The many shallow cracks interact in a complex manner where most cracks remain surface cracks but some cracks may propagate to become through-wall cracks and cause pipe failure.

This crack growth is primarily controlled by the complex thermal loadings but it also affected by the crack interactions and the shape of the cracks [6]; due to the stress gradients thermal fatigue cracks have generally a very large crack aspect ratio. The response of components to thermal loads depends on the heat transfer process. In certain components the pipe wall does not respond to high frequency fluctuation of the fluid temperature because of heat transfer loss; it depends also on the frequencies and on the time it takes for the heat to be conducted through the pipe. If the period ($1/f$) is much shorter than the heat conduction time then the temperature variations are confined to the surface, even when the heat transfer coefficient is high. On the other hand low frequency fluctuations may not cause significant thermal stresses because of thermal homogenization [7–9]. In between there is a frequency range in which thermal fatigue damage can occur. For instance numerical simulations of thermal striping at tee junctions of piping systems which have experienced such problems indicate that the oscillation frequency of surface temperature is in the range of 0.1–1.0 Hz [10–12].

In general thermal loading due to mixing occurs as a spectrum, the characteristics of which are not well known and cannot be reliably measured or calculated. This lack of knowledge of the loading constitutes the main complexity for a reliable analysis of the thermal fatigue life. The so-called sinusoidal method is a relatively simple model to assess the fatigue life for complex load spectra where only the nominal maximum temperature variations are known. The basic steps followed are outlined in Fig. 1. The temperature variation is assumed to be sinusoidal with a prescribed amplitude but unknown frequency. The associated

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Nomenclature

a_i	initial crack depth
a_f	final crack depth
$2c$	crack length
$2c/a$	crack shape ratio
l	wall-thickness of the pipe
r_i, r_o	inner and outer radii of the pipe
θ	temperature change
T_0	reference temperature
r	radial distance
κ	thermal diffusivity
ρ	mass density
c_{heat}	specific heat coefficient
$q(t)$	thermal boundary condition
λ	thermal conductivity
θ_0	amplitude of temperature wave
ω	wave frequency in rad/sec
f	wave frequency in Hz
σ_{rr}	radial stress
$\sigma_{\theta\theta}$	hoop stress
σ_{zz}	axial stress
σ_y	yield strength
E	Young's modulus
σ_{ts}	tensile strength
α	coefficient of the linear thermal expansion
ν	Poisson's ratio
x	radial local coordinate
σ_i	coefficients for stress distribution
G_i	influence coefficients
S_a	effective stress amplitude
D_{init}	duration of the crack initiation
N_p	number of cycles
D_{cg}	duration of the crack propagation
$\frac{da}{dN}$	crack growth rate
C	Paris law scaling parameter
n	Paris law exponent
K_I	Mode I stress intensity factor
R	stress intensity factor ratio

stresses in the hoop and axial direction are determined for this loading. In the previous works [13,14] a set of analytical solutions was developed for thermal stresses generated in a hollow cylinder under sinusoidal thermal loading based on the finite Hankel transform, some properties of Bessel's function and the thermo-elasticity governing equations. These solutions in turn allow the derivation of the associated elastic thermal stresses and also their distribution through the wall-thickness. The crack initiation life and the crack propagation life can then be determined. The crack initiation life is computed from the surface stress with the loading frequency that gives the shortest lifetime according to the relevant fatigue curves [10]. The procedure for calculating the crack propagation life is more complicated but the basic idea is similar. Stress intensity factors are derived from the stress profiles using handbook solutions. The crack propagation life is then computed using Paris law and the critical frequency is then defined as the frequency that gives the shortest life.

In present work, the Civaux 1 case [1] is used to illustrate the application of the sinusoidal method and the analytical thermal stress solutions for the crack initiation and crack growth life assessment in high cycle domain. Because the crack aspect ratio is known to be a critical input to the fatigue crack growth calculations, the paper gives detailed consideration to this issue. The crack profiles observed in the field failures [1], [6,7] show that fatigue cracking quite often results in relatively long flaws with large aspect ratio. Considering also other results reported in the literature [1,6] we suggest a range of crack aspect ratios which should be checked to ensure this effect is addressed in a conservative way for thermal fatigue flaw assessment. Finally, it is noted that the work is intended to support a proposed European Thermal Fatigue Procedure [10] for high cycle fatigue damage assessment in mixing tees.

2. Thermal stresses developed under sinusoidal thermal loading in pipes

The sinusoidal method requires that stresses and stress intensity factors are computed for a sufficiently broad frequency range. This in turn calls for computationally fast methods for reliable stress analysis. In previous works [13,14] analytical solutions were developed for temperature fields and associated elastic thermal stress distributions for a hollow cylinder assuming a sine-wave thermal loading. Since these form the basis of fatigue crack initiation and growth assessments, some of the main features are summarized below (see also Appendices A and B).

2.1. Temperature distribution

A pipe is assumed to be made of a homogeneous isotropic material with inner and outer radii r_i and r_o, respectively. The one-dimensional heat diffusion equation in cylindrical coordinates is given by:

$$\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}=\frac{1}{\kappa}\frac{\partial\theta}{\partial t},\quad (r_i\le r\le r_o, t\ge 0) \tag{1}$$

where $\theta = T(r,t)$ is the temperature change from the reference temperature at any position r and at time t. The reference temperature, T_0, is the temperature of the body in the stress free state, r is radial distance, κ is the thermal diffusivity which is defined as $\kappa=\lambda/(\rho\cdot c_{heat})$ (with λ thermal conductivity, ρ mass density, c_{heat} specific heat coefficient). The solution must satisfy the boundary conditions:

$$\theta(r_i,t) = q(t),\quad (t\ge 0) \tag{2}$$

$$\theta(r_o,t) = 0,\quad (t\ge 0) \tag{3}$$

and the initial condition:

$$\theta(r,0) = 0,\quad (r_i\le r\le r_o) \tag{4}$$

The function $q(t)$ is a function of time representing the thermal boundary condition applied on the inner surface of the cylinder. Using the finite Hankel transform methodology in the way developed in Refs. [13,14] (see also Appendix A), we are able to obtain the temperature distribution through the wall-thickness. If the boundary condition for the thermal loading at the inner surface is expressed as:

$$g(t) = \theta_0\sin(\omega t) = \theta_0\sin(2\pi\cdot f\cdot t), \tag{5}$$

then the radial temperature distribution through the wall-thickness of the hollow cylinder is derived from Eq. (A-8) as: