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Discrete singular convolution method for the analysis of Mindlin plates on elastic foundations

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Abstract

The method of discrete singular convolution (DSC) is used for the bending analysis of Mindlin plates on two-parameter elastic foundations for the first time. Two different realizations of singular kernels, such as the regularized Shannon's delta (RSD) kernel and Lagrange delta sequence (LDS) kernel, are selected as singular convolution to illustrate the present algorithm. The methodology and procedures are presented and bending problems of thick plates on elastic foundations are studied for different boundary conditions. The influence of foundation parameters and shear deformation on the stress resultants and deflections of the plate have been investigated. Numerical studies are performed and the DSC results are compared well with other analytical solutions and some numerical results.

Keywords: Discrete singular convolution; Mindlin plate; Bending; Pasternak foundations; Winkler foundation

1. Introduction

As parallel to the computer technology during the past 50 years, numerical methods have made significant progress. The discrete singular convolution (DSC) method proposed by Wei [1] in 1999 is a relatively new numerical discretization technique for approximation of derivatives. The DSC method has been successfully applied to solve governing differential equations for various boundary value and eigenvalue problems in engineering [3–4], structural mechanics [5–12], and fluid mechanics [13], by this time. Plates on elastic foundation have wide applications in pressure vessels technology such as petrochemical, marine and aerospace industry, civil, and mechanical engineering. A number of analytical and numerical studies have been conducted on the static and dynamic analysis of plates on elastic foundations. Long list of references on dynamic and bending analysis of thin and thick plates on elastic foundation are given, for example, in Refs. [14,15,20,24]. Some selected works in this research topic includes those of Liew et al. [16], Teo and Liew [17], Wang et al. [18], Kobayashi and Sonoda [19].

In the present study, a numerical method is developed for the static analysis of Mindlin plates on two-parameter elastic foundations. The procedure is based on the application of the DSC method. To the authors' knowledge, this is the first time the DSC method has been successfully applied to Mindlin plate on elastic foundation problems for the analysis of bending. The organization of the paper is as follows. DSC algorithm is briefly presented in Section 2. Theory and related formulations of Mindlin plate on elastic foundation are given in Section 3. The application of this algorithm to title problem is given in Section 4. Conclusions are given in Section 5.

2. Discrete singular convolution (DSC)

The DSC method was originally introduced by Wei [1] as a simple and highly efficient numerical technique. Like some other numerical methods, the DSC method discretizes the spatial derivatives and, therefore, reduces the given partial differential equations into a standard eigenvalue problem. For brevity, consider a distribution,

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T and $\eta(t)$ as an element of the space of the test function. A singular convolution can be defined by [2]

$$F(t) = (T*\eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)\,\mathrm{d}x, \tag{1}$$

where $T(t-x)$ is a singular kernel. The DSC algorithm can be realized by using many approximation kernels. However, it was shown [3–10] that for many problems, the use of the regularized Shannon kernel (RSK) is very efficient. The RSK is given by [6]

$$\delta_{\Delta,\sigma}(x-x_k) = \frac{\sin[(\pi/\Delta)(x-x_k)]}{(\pi/\Delta)(x-x_k)} \times \exp\left[-\frac{(x-x_k)^2}{2\sigma^2}\right]; \quad \sigma>0, \tag{2}$$

where $\Delta = \pi/(N-1)$ is the grid spacing and N is the number of grid points. The parameter σ determines the width of the Gaussian envelope and often varies in association with the grid spacing, i.e., $\sigma = rh$. With a sufficiently smooth approximation, it is more effective to consider a DSC [4]

$$F_\alpha(t) = \sum_k T_\alpha(t-x_k) f(x_k), \tag{3}$$

where $F_\alpha(t)$ is an approximation to $F(t)$ and $\{x_k\}$ is an appropriate set of discrete points on which the DSC of Eq. (1) is well defined. In the DSC method, the function $f(x)$ and its derivatives with respect to the x coordinate at a grid point x_i are approximated by a linear sum of discrete values $f(x_k)$ in a narrow bandwidth $[x-x_M,\ x+x_M]$. This can be expressed as [5]

$$\left.\frac{\mathrm{d}^n f(x)}{\mathrm{d}x^n}\right|_{x=x_i} = f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta^{(n)}_{\Delta,\sigma}(x_i - x_k) f(x_k); \quad (n = 0,1,2,\ldots,), \tag{4}$$

where superscript n denotes the nth-order derivative with respect to x. The x_k is a set of discrete sampling points centered around the point x, σ is a regularization parameter, Δ is the grid spacing, and $2M+1$ is the computational bandwidth, which is usually smaller than the size of the computational domain. The higher order derivative terms $\delta^{(n)}_{\Delta,\sigma}(x-x_k)$ in Eq. (4) are given as below [7]:

$$\delta^{(n)}_{\Delta,\sigma}(x-x_k) = \left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^n [\delta_{\Delta,\sigma}(x-x_k)], \tag{5}$$

where the differentiation can be carried out analytically. The discretized forms of Eq. (5) can then be expressed as

$$f^{(n)}(x) = \left.\frac{\mathrm{d}^n f}{\mathrm{d}x^n}\right|_{x=x_i} \approx \sum_{k=-M}^{M} \delta^{(n)}_{\Delta,\sigma}(k\Delta x_N) f_{i+k,j}, \tag{6}$$

when the regularized Shannon's delta (RSD) kernel is used, the detailed expressions for, $\delta^{(n)}_{\Delta,\sigma}(x)$ can be easily obtained. Detailed formulations for these differentiation coefficients can be found in Refs. [2,3]. For example, second-order derivative is given as

$$\begin{aligned}
\delta^{(2)}_{\pi/\Delta,\sigma}(x_m - x_k) &= -\frac{(\pi/\Delta)\sin(\pi/\Delta)(x-x_k)}{(x-x_k)}\exp[-(x-x_k)^2/2\sigma^2] \\
&\quad - 2\frac{\cos(\pi/\Delta)(x-x_k)}{(x-x_k)^2}\exp[-(x-x_k)^2/2\sigma^2)] \\
&\quad - 2\frac{\cos(\pi/\Delta)(x-x_k)}{\sigma^2}\exp[-(x-x_k)^2/2\sigma^2] \\
&\quad + 2\frac{\sin(\pi/\Delta)(x-x_k)}{\pi(x-x_k)^3/\Delta}\exp[-(x-x_k)^2/2\sigma^2)] \\
&\quad + \frac{\sin(\pi/\Delta)(x-x_k)}{\pi(x-x_k)\sigma^2/\Delta}\exp[-(x-x_k)^2/2\sigma^2] \\
&\quad + \frac{\sin(\pi/\Delta)(x-x_k)}{\pi\sigma^4/\Delta}(x-x_k)\exp[-(x-x_k)^2/2\sigma^2)].
\end{aligned} \tag{7}$$

At $x = x_k$, this derivative is given by

$$\delta^{(2)}_{\pi/\Delta,\sigma}(0) = -\frac{1}{3}\frac{3+(\pi^2/\Delta^2)\sigma^2}{\sigma^2} = -\frac{1}{\sigma^2} - \frac{\pi^2}{3\Delta^2}. \tag{8}$$

Another important kernel is the Lagrange kernel. The differentiation in Eq. (5) can also be easily carried out for a finite Lagrange kernel

$$\delta_{\Delta,\sigma}(x) = \prod_{i=-M,k\neq i}^{M} \frac{x-x_i}{x_k-x_i}. \tag{9}$$

In this case, the first- and second-order derivatives are given as

$$\delta^{(1)}_{\Delta,\sigma}(x) = \sum_{i=-M;i\neq k}^{M}\left(\frac{1}{x_k-x_i}\right)\prod_{i=-M,k\neq i}^{i+M}\frac{x-x_i}{x_k-x_i}, \tag{10}$$

$$\delta^{(2)}_{\Delta,\sigma}(x) = \sum_{\substack{i,m=-M;\ i\neq k\\ m\neq k,\ i\neq m}}^{M}\left(\frac{1}{(x-x_i)(x-x_m)}\right)\prod_{i=-M,\ k\neq i}^{i+M}\frac{x-x_i}{x_k-x_i}. \tag{11}$$

3. Fundamental equations of bending

In the present study, the foundation is modeled in terms of Winkler parameter K and shear parameter G_f of the Pasternak model [25,26]. The governing equations for bending of Mindlin plates on two-parameter elastic foundation can be given as [17]

$$D\left[\frac{1-\upsilon}{2}\left(\frac{\partial^2\psi_x}{\partial x^2}+\frac{\partial^2\psi_x}{\partial y^2}\right)+\frac{1+\upsilon}{2}\left(\frac{\partial^2\psi_x}{\partial x^2}+\frac{\partial^2\psi_x}{\partial x\partial y}\right)\right] + \kappa Gh\left(\frac{\partial w}{\partial x}-\psi_x\right) = 0, \tag{12a}$$

$$D\left[\frac{1-\upsilon}{2}\left(\frac{\partial^2\psi_y}{\partial x^2}+\frac{\partial^2\psi_y}{\partial y^2}\right)+\frac{1+\upsilon}{2}\left(\frac{\partial^2\psi_x}{\partial x\partial y}+\frac{\partial^2\psi_y}{\partial x\partial y}\right)\right] + \kappa Gh\left(\frac{\partial w}{\partial y}-\psi_y\right) = 0, \tag{12b}$$