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Comparison of linearized vs. non-linearized multibody vehicle model for real-time simulation

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ABSTRACT

The performance of a real-time multibody dynamics simulation using linearized constraint equations is discussed. It is known that a small step size is required to simulate a system with very stiff elements such as rubber bushes or a system with high frequency phenomena. Calculation time can be reduced by linearizing constraint equations, and high-speed multibody dynamic analysis with a smaller step size is realized. It is important to acknowledge the degree of accuracy lost when linearized constraint equations are used. We compare linearized and non-linearized simulation models using a model of an actual vehicle and ISO-8608 road profile classifications. Calculation time is evaluated in a real-time analysis environment, and the accuracy of the simulation result is examined.

1. Introduction

Multibody dynamics is an effective approach to reduce the cost and the development time of mechanical products. A typical application of multibody dynamics in automotive engineering is the modeling and analysis of suspension systems [1]. Multibody vehicle models allow precise pre-prototype evaluation of the effects of suspension geometry and mechanical characteristics of springs, bushes, and dampers on the handling performance and ride comfort of a vehicle.

In reports published as early as the 1980s, the use and advantages of real-time analysis of multibody dynamics have been discussed in several articles [2,3]. Cuadrado et al. [4] addressed how the modeling process affects the dynamic simulation of multibody systems and how it could be used to define the concept of intelligent simulation. With improvements in CPU performance, the computational speed of multibody dynamics has been improved, and several real-time multibody dynamics simulation applications, such as driving simulators and hardware-in-the-loop simulation (HILS) systems have also proposed [5–8].

However, enhancing the speed of real-time analysis is in strong demand. It is well known that the compliance effects of rubber bushes in a suspension subsystem are integral to quality handling and ride characteristics of an automobile. Due to their high stiffness, numerical integration analysis of the effects of rubber bushes requires a very small step size. Therefore, it is difficult to simulate the effect of rubber bushes in HILS or driving simulator systems adequately in real-time. Kim proposed a subsystem synthesis method to enhance the efficiency of real-time analysis, in which each subsystem was independently analyzed with a virtual reference body [9]. Kim also proposed a quasi-static analysis method to consider bush compliance effects for a real-time multibody vehicle model [10]. Uchida [11] used the theory of Gröbner bases to triangularize systems modeled with an arbitrary set of coordinates, which resulted in a system of equations that can be solved recursively. This theory was applied to the analysis of a 6-DOF Stewart–Gough platform. In addition, the use of matrix libraries for real-time analysis is a practical way to enhance calculation speed [12].

In this paper, the authors propose to apply linearized constraint equations to the real-time analysis of a multibody vehicle model to reduce calculation time. The proposed method is an approximated analysis method, and the degree of accuracy lost by the linearization of the constraint equations should be acknowledged. In this study, vehicle dynamics simulations on 3D road profiles were conducted using linearized and non-linearized multibody models, and the accuracy and computational speed were examined.

2. Multibody system analysis with linearized constraint equations

2.1. Multibody analysis with relative coordinates

The proposed analysis method with linearization is based on the fact that, when the equation of motion is described with relative coordinates, the variation of the Jacobian matrix of the

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constraint equations remains small as long as the relative motion of each body is small. In this section, details regarding the derivation of the equation of motion for a constrained system using relative coordinates are given.

The equation of motion for a multibody system using absolute Cartesian coordinates can be written as follows [2]:

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} & \boldsymbol{\Phi}_{\mathbf{r}}^T \\ \mathbf{0} & \mathbf{J}' & \boldsymbol{\Phi}_{\pi'}^T \\ \boldsymbol{\Phi}_{\mathbf{r}} & \boldsymbol{\Phi}_{\pi'} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{r}} \\ \dot{\omega}' \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{F}^A \\ \mathbf{n}'^A - \tilde{\omega}'\mathbf{J}'\omega' \\ \gamma \end{bmatrix}, \quad (1)$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{J}'$ is the inertia tensor, $\mathbf{r}$ is the position of the centers of gravity of each body, ω' is the angular velocity of a body-fixed frame, λ are the Lagrange multipliers, $\boldsymbol{\Phi}_{\mathbf{r}}$ and $\boldsymbol{\Phi}_{\pi'}$ are the Jacobian matrices of the constraints, $\mathbf{F}^A$ and $\mathbf{n}'^A$ are applied forces and torques, and γ is the acceleration equation. In the right side of the equation, $\tilde{\omega}' = \mathrm{diag}[\tilde{\omega}'_1\ \tilde{\omega}'_2\ \cdots\ \tilde{\omega}'_n]$, and

$$\tilde{\omega}'_i = \begin{bmatrix} 0 & -\omega'_{iz} & \omega'_{iy} \\ \omega'_{iz} & 0 & -\omega'_{ix} \\ -\omega'_{iy} & \omega'_{ix} & 0 \end{bmatrix}. \quad (2)$$

In the rigorous dynamic analysis of multibody systems, the coefficient matrix in the left term must be evaluated for each time step of the numerical integration. This process requires considerable calculation time as the number of bodies increases.

Next, the relative position $\mathbf{r}^*$ to the body-fixed coordinate of a body j, which is one of the bodies in the system of interest, is considered. The equation of motion can be rewritten as

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} & \boldsymbol{\Phi}_{\mathbf{r}^*}^{\mathbf{T}} \\ \mathbf{0} & \mathbf{J}' & \boldsymbol{\Phi}_{\pi^*}^{\mathbf{T}} \\ \boldsymbol{\Phi}_{\mathbf{r}^*} & \boldsymbol{\Phi}_{\pi^*} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{r}}^* \\ \dot{\omega}' \\ \lambda^* \end{bmatrix} = \begin{bmatrix} \mathbf{F}^{A*} + \mathbf{F}^I \\ \mathbf{n}'^A - \tilde{\omega}' J' \omega' \\ \gamma^* \end{bmatrix} \quad (3)$$

in which the Jacobian matrices $\boldsymbol{\Phi}_{\mathbf{r}^*}$ and $\boldsymbol{\Phi}_{\pi^*}$ and the acceleration equation γ^* are calculated using the relative position $\mathbf{r}^*$. In this equation, $\mathbf{F}^{A*}$ is the applied force of each body described in the body-fixed coordinate of the body j, and can be obtained as follows:

$$\mathbf{F}^{A*} = \begin{bmatrix} \mathbf{A}_j^T \mathbf{F}_1^A \\ \mathbf{A}_j^T \mathbf{F}_2^A \\ \vdots \\ \mathbf{A}_j^T \mathbf{F}_n^A \end{bmatrix}, \quad (4)$$

where $\mathbf{A}_j$ is the direction cosine matrix of the body j. The inertia force $\mathbf{F}^I$ should be considered in the equation of motion, which is composed of centrifugal, Coriolis, and other forces, by using the translational and rotational acceleration, etc., of the body j as follows:

$$\mathbf{F}^I = \begin{bmatrix} \mathbf{A}_j^T \mathbf{m}_1(-\ddot{\mathbf{r}}_j) - \mathbf{m}_1(\tilde{\dot{\omega}}'_j \mathbf{r}_1^* + \tilde{\omega}'_j \tilde{\omega}'_j \mathbf{r}_1^* + 2\tilde{\omega}'_j \dot{\mathbf{r}}_1^*) \\ \mathbf{A}_j^T \mathbf{m}_2(-\ddot{\mathbf{r}}_j) - \mathbf{m}_2(\tilde{\dot{\omega}} + j' \mathbf{r}_2^* + \tilde{\omega}'_j \tilde{\omega}'_j \mathbf{r}_2^* + 2\tilde{\omega}'_j \dot{\mathbf{r}}_2^*) \\ \vdots \\ \mathbf{A}_j^T \mathbf{m}_n(-\ddot{\mathbf{r}}_j) - \mathbf{m}_n(\tilde{\dot{\omega}}'_j \mathbf{r}_n^* + \tilde{\omega}'_j \tilde{\omega}'_j \mathbf{r}_n^* + 2\tilde{\omega}'_j \dot{\mathbf{r}}_n^*) \end{bmatrix}. \quad (5)$$

The relative motions between the body j and other bodies can be considered with Eqs. (3)–(5). Finally, the position $\mathbf{r}_j$ of the body j in absolute coordinates is obtained with the following equation:

$$m_j \ddot{\mathbf{r}}_j = m_j \begin{bmatrix} \ddot{x}_j \\ \ddot{y}_j \\ \ddot{z}_j \end{bmatrix} = \mathbf{F}_j^A + \mathbf{A}_j \mathbf{F}_j^{C*}. \quad (6)$$

In this equation, $\mathbf{F}_j^{C*}$ is the constraint force on the body j, which is a part (submatrix) of the constraint force

$$\mathbf{Q}^{C*} = \begin{bmatrix} \mathbf{F}^{C*} \\ \mathbf{n}'^C \end{bmatrix} = -\boldsymbol{\Phi}_{\mathbf{q}^*}^T \lambda^*, \quad (7)$$

where $\boldsymbol{\Phi}_{\mathbf{q}^*} = [\boldsymbol{\Phi}_{r^*} \boldsymbol{\Phi}_{\pi^*}]$.

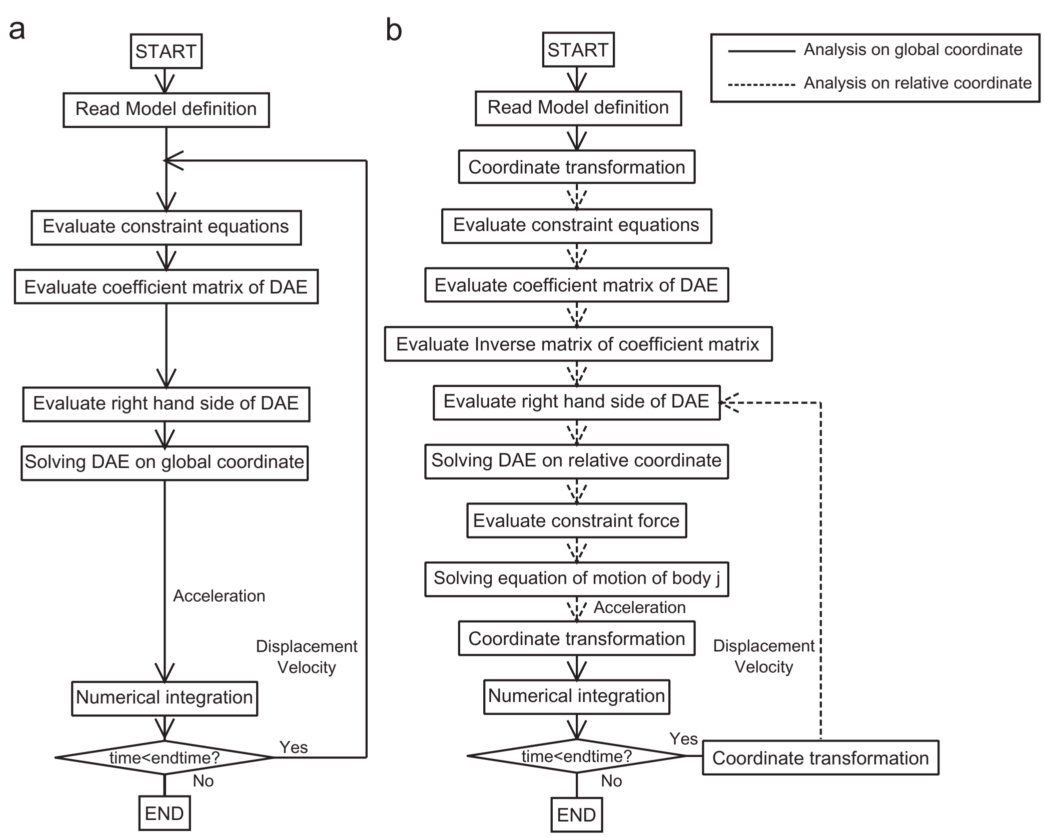

Fig. 1. Multibody dynamics analysis flow charts. (a) Exact multibody dynamic analysis and (b) proposed multibody dynamic analysis.