International Journal of Non-Linear Mechanics 47 (2012) 851–862

Contents lists available at SciVerse ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Rigid bodies formulation arising from visco-plastic flows with some numerical illustrations

J. Ferchichi [a,b,*], H.B. Khenous [b,c], J.P. Zolésio [d,e]

[a] Research Unity: ACPDE, Science Faculty of Monastir, Tunisia
[b] King Khalid University, Saudi Arabia
[c] LMS, Ecole Polytechnique, France
[d] OPALE-INRIA Sophia-Antipolis, France
[e] CNRS, France

ARTICLE INFO

Article history:
Received 26 October 2011
Received in revised form
25 April 2012
Accepted 26 April 2012
Available online 30 May 2012

Keywords:
Visco-plastic flows
Norton–Hoff model
Free interfaces
Rigid body

ABSTRACT

This work reveals a weak formulation of rigid bodies motion arising from free boundaries evolution in visco-plastic flows. This phenomena could modelize the cooling of blood-plasma in thin arteries. The fluid motion is governed by the incompressible Norton–Hoff operator in the non-cylindrical quasi-static case coupled with the time-dependent convection system. We supply the existence of an interface between two non-miscible fluids by the use of the non-smooth evolution theory. We prove that the fluid's flow is transformed to a rigid body when its consistency is very large. The established result is, in some sense, an extension of the existing formulations. We supply some numerical illustrations to validate our results.

© 2012 Elsevier Ltd. All rights reserved.

1. Introduction and motivation

We deal with rigid bodies motion connected to non-smooth evolutions of visco-plastic flows. The fluid's motion is governed by the incompressible quasi-static Norton–Hoff model, whose the coefficients are non-cylindrical, coupled with the time-dependent convection system. The Norton–Hoff operator is inherently non-linear. The main purpose of this work is to reveal the relationship between a rigid body, established of visco-plastic materials, immersed in visco-plastic flow and a created free interface between two non-miscible flow solutions to the system (1)–(4), (6). The principal results are argued in Theorems 2.3 and 3.1.

The main difficulties in this analysis are:

1. The coupling between the velocity and the moving domain involves non-cylindrical coefficients.
2. The flow's velocity is not regular enough which does not allow to adopt the tools of the classical evolution.
3. The compactness in time and space is required but we have no estimate on $\partial_t u$.

* Corresponding author at: Research Unity: ACPDE, Science Faculty of Monastir, Tunisia. Tel.: +216 73 500 280; fax: +216 73 500 278.
E-mail addresses: jamel.ferchichi@fsm.rnu.tn (J. Ferchichi), khenous@kku.edu.sa (H.B. Khenous), jean-paul.zolesio@sophia.inria.fr (J.P. Zolésio).

0020-7462/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijnonlinmec.2012.04.011

In recent years, many papers concerning the study of rigid body motion in either incompressible or compressible flows governed mainly by the Navier–Stokes equations have been published in the literature (see [3–5,11,13]). Temam has proven that velocities describe rigid body motion if and only if their strain tensor vanishes in the classical sense (see [14]). In this work, we extend the Temam-formulation to visco-plastic flows, governed by the quasi-static Norton–Hoff model (see [9,8]), in a *weak* sense by treating the phenomena of free interfaces in visco-plastic flows which has never been studied.

The paper is organized as follows. In Section 2, we first stand the model that describes the problematic. Second, we treat the phenomena of free interfaces between two non-miscible Norton–Hoff fluids in the quasi-static case coupled with the time-dependent convection equations. Then the existence result to the considered problem is argued. It is based on the non-smooth theory of evolution technics coupled with the Schauder fixed point theorem applied to an established iterative process. Moreover, we prove the compactness of the solutions set. The next section is devoted to the formulation of the rigid body motion. We establish that a rigid body motion is reached when the consistency of the fluid occupying the domain is very large which recovers the Temam-formulation in a weak sense. In the last section, we validate our results with numerical examples in two cases with respect to the values of the fluid consistency α either when it is small or large.

2. Free interfaces in visco-plastic flows

We consider two fluids of Norton–Hoff type with the same exponent p occupying a fixed region D of $\mathbb{R}^N, N=2$ and undergoing the action of time-dependent volume source f. We assume that the mapping of the exterior exertion is sufficiently slow in order to neglect the inert effects. Then the evolution is quasi-static. The fluids are supposed non-miscible and we denote by α and by β their consistencies.

2.1. Two phases model of Norton–Hoff problem

The evolution, on a time interval $I=(0,T)$, is a solution of the following non-linear problem of transmission formulated in the cylinder $I \times D$:

$$\begin{cases} K(t,x)|\varepsilon(u)|^{p-2}\varepsilon(u)-PId=\sigma & \text{in } I \times D, \\ -\text{div}(\sigma)=f & \text{in } I \times D, \\ \text{div}(u)=0 & \text{in } I \times D, \end{cases} \tag{1}$$

with the homogeneous Dirichlet boundary conditions

$$u=0 \quad \text{on } I \times \partial D, \tag{2}$$

and let consider the following initial state:

$$u(0,.)=u_0(\cdot) \quad \text{in } D, \tag{3}$$

where D denotes a smooth bounded open subset, locally on one side of its C^2-boundary ∂D, σ is the Cauchy stress tensor, $\varepsilon(u)=\frac{1}{2}(\nabla(u)+\nabla(u)^*)$ is the linearized strain velocity tensor, ∇ is the gradient operator, p is the exponent of the material; $1<p<2$: it is the sensibility coefficient of the material to the strain velocity tensor, P is the hydrostatic pressure, Id is the identity tensor, f is an external force acting on the fluid and u_0 is a given data. The first equation designates the behavior law, the second describes the balance state and the third one prescribes the incompressibility of the fluid during the evolution. The coefficient $K(t,x)$ is linked to the consistencies of the fluids at the instant t on the position x. Let

$$K(t,x)=\alpha(t)+\beta(t),$$

$$\alpha(t)=\alpha\chi_{\Omega_t},$$

$$\beta(t)=\beta\chi_{\overline{\Omega_t}^c}, \tag{4}$$

where Ω is a bounded subdomain of D occupied by the fluid of consistency α at the initial time, and, thus $K(t,x)$ is a non-smooth function with respect to the moving domain Ω_t with complementary Ω_t^c which makes sense to the existence of the free boundary. Notice that $\Omega=\Omega(t=0)$ is assumed to be locally on one side of its C^2-boundary $\partial\Omega$.

Our point of view is to consider the solution as an evolution of the measurable-domain Ω and, thus an evolution of the interface between the two non-miscible fluids. We look for a solution to the system (1)–(4) as a Schauder fixed point of an established operator via a *virtual* evolution of the interface with a velocity u_V. The velocity field u_V is the solution of the Norton–Hoff model with respect to a fixed velocity V. We may use evolution technicalities since we deal with a transmission problem.

2.2. Functional setting

Let us introduce the adequate functional framework in space,

$$\mathcal{W}=W_0^{1,p}(\Omega,\mathbb{R}^N)^3, \quad \mathcal{W}_{div}=\{v\in\mathcal{W} \text{ s.t. div}(v)=0 \text{ on } \Omega\},$$
$$L_0^2(\Omega)=L^2(\Omega)/\mathbb{R}.$$

Further, we introduce the set of the admissible right-hand sides,

$$\mathcal{W}'=W^{-1,p'}(\Omega,\mathbb{R}^N)^3,$$

which is the topological dual space of $\mathcal{W}$ endowed with its natural norm, where p' is the algebraic dual of p. We define $\langle \cdot,\cdot \rangle = \langle \cdot,\cdot \rangle_{V^*,V}$ with V^* is the topological dual space of V. All the other functional spaces used with their natural structure unless the mentioned.

2.3. Preliminaries

The following result provides a Banach space structure for the spaces $\mathcal{W}$ and $\mathcal{W}_{div}$.

Proposition 2.1. *The mapping* $\|\cdot\|$ *defined from* $\mathcal{W}$ *to* $\mathbb{R}^+$ *by*

$$\|v\|=\left(\int_\Omega|\varepsilon(v)|^p\right)^{1/p}$$

is a norm on $\mathcal{W}$ *equivalent to the usual one* (*see* [7,10]), *where* $|\cdot|$ *means the Euclidean norm of matrices in* $\mathbb{R}^{N^2}$.

Definition 2.1. We introduce the Norton–Hoff functional or the so-called compliance functional given by

$$\Phi_\Omega:\mathcal{W}(\Omega)\longrightarrow\mathbb{R}$$

$$v\longrightarrow\int_\Omega\frac{K_c}{p}|\varepsilon(v)|^p-fv. \tag{5}$$

The existence and uniqueness of a solution to the Norton–Hoff problem $\mathcal{P}$ is derived from the properties of Φ_Ω.

Proposition 2.2. *The functional* Φ_Ω *is strictly convex, weakly l.s.c., Gateaux differentiable and coercive.*

The well-posedness of the static Norton–Hoff problem is supplied by the following result. The proof given in [7] is based on the properties of the compliance functional.

Theorem 2.1. *The Norton–Hoff mixed problem in the static case has a unique solution* (u,P) *in* $\mathcal{W}\times L_0^2$.

Remark 2.1. The Norton–Hoff model is not regular enough, then we are not able to use the classical theory of evolution in which the flow mapping has to be associated to a vector field whom the space-regularity is, at least, Lipschitzian. Nevertheless, non-smooth evolution will be considered. We will use some new technics of the speed method for shape analysis that were introduced in [1] and extended in [16].

Knowing that Sobolev functions cannot have discontinuities along hypersurfaces, as, on the contrary, is required by the model above, the Sobolev space analysis is then no longer appropriate for this kind of problems. For a rigorous presentation, the main tool is the spac BV.

2.4. Non-smooth evolution of domains

The space of bounded variations is introduced by

$$BV(D)=\{v\in L^1(D) \text{ s.t. } \nabla v\in\mathcal{M}^1(D)\},$$

where $\mathcal{M}^1(D)$ is the Banach space of bounded measures endowed with its natural norm.

We set

$$\mathcal{E}^{1,1}=\{v\in L^1(I,D)^3 \text{ s.t. div } v\in L^1(I,D) \text{ and } v\cdot n=0 \text{ in } W^{-1,1}(\partial D)\},$$

where n is the outward normal unit vector of ∂D.

We denote by

$$\mathcal{V}=\mathcal{E}^{1,1}\cap\{v\in L^1(I,BV(D)^3) \text{ s.t. div}(v)_-, \text{ div}(v)_+\in L^1(I,L^\infty(D))\},$$

where $\text{div}(v)_+$ (respect. $\text{div}(v)_-$) is the positive (respect. negative) part of $\text{div}(v)$.

Download English Version:

https://daneshyari.com/en/article/787989

Download Persian Version:

https://daneshyari.com/article/787989

Daneshyari.com