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Mathematical modeling of axial flow between two eccentric cylinders: Application on the injection of eccentric catheter through stenotic arteries

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ABSTRACT

This study is concerned with the surgical technique for the injection of catheter through stenotic arteries. The present theoretical model may be considered as mathematical representation to the movement of physiological fluid representing blood in the gap between two eccentric tubes (eccentric-annulus flows) where the inner tube is uniform rigid representing moving catheter while the other is a tapered cylindrical tube representing artery with overlapping stenosis. The nature of blood is analyzed mathematically by considering it as a Newtonian fluid. The analysis is carried out for an artery with a mild stenosis. The problem is formulated using a perturbation expansion in terms of a variant of the eccentricity parameter (the parameter that controls the eccentricity of the catheter position) to obtain explicit forms for the axial velocity, the stream function, the resistance impedance and the wall shear stress distribution also the results were studied for various values of the physical parameters, such as the eccentricity parameter ϵ, the radius of catheter σ, the velocity of catheter V_o, the angle of circumferential direction θ (azimuthal coordinate), the taper angle ϕ and the maximum height of stenosis δ^*. The obtained results show that there is a significant deference between eccentric and concentric annulus flows through catheterized stenosed arteries.

1. Introduction

The flow of the fluid in pipes of different shapes is important in many biological and biomedical systems, the human cardiovascular system and in several technological devices. The abnormal and unnatural growth in the lumen of an artery is called stenosis. Localized atherosclerotic constrictions in arteries (arterial stenosis) are found predominantly in the internal carotid artery, which supplies blood to the brain, the coronary artery, which supplies blood to the cardiac muscles and the femoral artery, which supplies blood to the lower limbs. In medicine, catheterization refers to a procedure in which a long, thin, flexible plastic tube (catheter) can be inserted or injected into a body cavity, duct or vessel. The catheter procedure thereby allow diagnose and treat heart and blood vessel conditions. One or more of catheters can be injected by a syringe through a peripheral blood vessel in the arm (antecubital artery or vein) or leg (femoral artery or vein) in the presence of x-ray guidance. This procedure gathers information such as adequacy of blood supply through the coronary arteries, blood pressure, blood flow throughout chambers of the heart, collection of blood samples and x-rays of the heart's ventricles or arteries. The vascular system may be injected by a dye to determine any blockages, narrowing or abnormalities in the coronary arteries. By using x-rays guidance appears some visible signs which assess the patient's need and his readiness for surgery, or perhaps a less invasive approach, such as dilation of a narrowed blood vessel either surgically or with the use of a balloon (angioplasty). The insertion of a catheter in an artery will form an annular region between the walls of the catheter and artery. This will alter the flow field, modifying the pressure distribution and increase the resistance. Hence, the pressure or pressure gradient recorded by a transducer attached to the catheter will differ from that of an non-catheterized artery. Therefore, it is important to study the effect due to the presence of catheter in the physiological artery flows.

Another model in the surgical process known as thread injection is a newly devised surgical technique which enables porous medical implants to be placed inside the body in a minimally invasive way, thus reducing surgical trauma. The thread is stored on a spool and injected within a fluid by applying an axial pressure gradient to the cylindrical container holding the liquid and the thread. The thread velocity is controlled by a motor. It is clearly desirable for the thread to be injected smoothly and to not suffer lateral deviation: it is therefore important that the flow is kept laminar [1]. The injection of fluid

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into the body using a needle or syringe is an important application of fluid dynamics. Through modeling the problem mathematically, a better understanding of the way in which key parameters, such as the speed of injection, affect the fluid flow characteristics within the syringe can be obtained. It is hoped that with this investigations, the injection can be carried out more proficiently and the pain of the patient can be reduced. A particular application of chemical thread injection is in lip augmentation found in most plastic surgery industries [2] (Fig. 1).

The flow between eccentric cylinders continues to attract considerable interest because of its immense theoretical and technical importance. The practical significance of this flow arises from the fact that it occurs in journal bearings which are widely used for hydrodynamic lubrication [3]. A simplified version of the injection process can be modeled mathematically by considering the axial flow between concentric cylinders with the inner cylinder (representing the thread or catheter) moving at a constant velocity. The first study was that of Frei et al. [4], who examined the problem experimentally and theoretically. They modeled the thread injection process by using a cylindrical rubber filament to represent the catheter. The filament was allowed to move concentrically through a steel cylindrical pipe (representing the injection vessel) filled with water. They measured various quantities including the axial force on the thread due to pressure gradient and viscous effects. On comparing the results with the theoretical predictions that arise from the exact Navier–Stokes solution for axial flow between concentric cylinders they discovered that the force measured in the experiments was always significantly less than that predicted by their theory. This observation forms the motivation for the current study in which we investigate whether this discrepancy could be caused by a non-linear instability of the basic thread-annular flow. Walton [5] considered the stability of the basic thread-annular flow and reported that there are discrepancies between the theoretical and experimental results. All previous theoretical studies assume that the thread to remain in a concentric position.

The important contributions of recent years to the topic are referenced in the literature [6–10]. Recently, much numbers of researchers [11–18] studied the problem of two-dimensional flow for Newtonian or non-Newtonian fluids (as a blood model) through two concentric tubes where the inner tube represents catheter while the outer tube represents stenotic artery. Realistically, the study may be very interesting in three-dimensions by considering the catheter in a eccentric position. No attempts have been made yet to discuss the eccentric- annulus flow through catheterized stenosed arteries. Therefore the main purpose of the present paper is to discuss the mathematical representation for the injection of eccentric catheter through stenotic arteries. A motivation of the present analysis is the hope that such a problem will be applicable in many clinical applications. The problem is first modeled and the non-dimensional governing equations are formulated. The non-dimensional governing equations in the case of mild stenosis and the corresponding boundary conditions are prescribed then solved analytically for the axial velocity. The results for the resistance impedance and wall shear stress distribution have been discussed for various values of the problem parameters. Also the contour plots for the stream function is discussed. Finally, the main finding of the results are summarized as a concluding remarks.

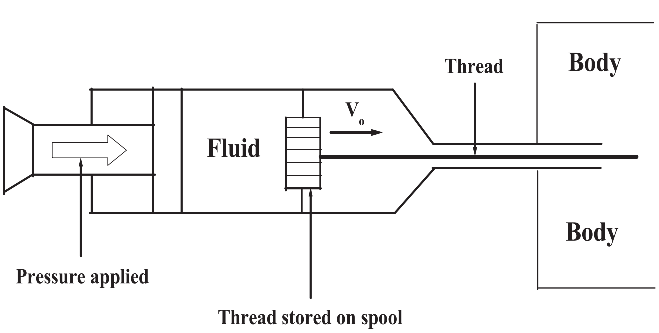

Fig. 1. The sketch of syringe shows the process of thread or catheter—injection.

2. Formulation of the problem

Let (r,θ,z) be the coordinates of a material point in the cylindrical polar coordinate system where the z-axis is taken along the axis of the artery while r, θ are along the radial and circumferential directions, respectively. Consider a viscous incompressible Newtonian fluid of viscosity μ and density ρ flowing through two eccentric tubes where the outer tube having a finite length L representing artery with overlapping stenosis while the inner tube is uniform rigid representing catheter of radius σ placed in an eccentric position within the artery. The center of catheter is now at position $(z=0, y=0)$ where y and z are coordinates in the cross-section of the pipe as shown in Fig. 2. The boundary of the catheter is described to order ϵ by $r=\sigma+\epsilon\cos(\theta)$ (obtained by using the cosine rule) where $\epsilon \ll \sigma$ is the parameter that controls the eccentricity of the catheter position. If $(\epsilon=0)$ then the catheter is in a concentric position [2]. The catheter is moving in the axial direction with velocity V_o. The geometry of the arterial wall with time-variant overlapping stenosis is defined by the function $R(z,t)$ as shown in Fig. 2 can be written mathematically as [19]

$$\begin{aligned} R(z,t) &= \left[(mz+R_o)-\frac{\delta\cos\phi}{L_o}(z-d)\left\{11-\frac{94}{3L_o}(z-d)+\frac{32}{L_o^2}(z-d)^2 - \frac{32}{3L_o^3}(z-d)^3\right\}\right]\Omega(t), \quad d\le z\le d+\frac{3L_o}{2} \\ &= (mz+R_o)\Omega(t) \quad \text{otherwise} \end{aligned} \tag{1}$$

where $R(z,t)$ denotes the radius of the tapered arterial segment in the constricted region, R_o is the constant radius of the normal artery in the non-stenotic region, ϕ is the angle of tapering, $3L_o/2$ is the length of overlapping stenosis, d is the location of the

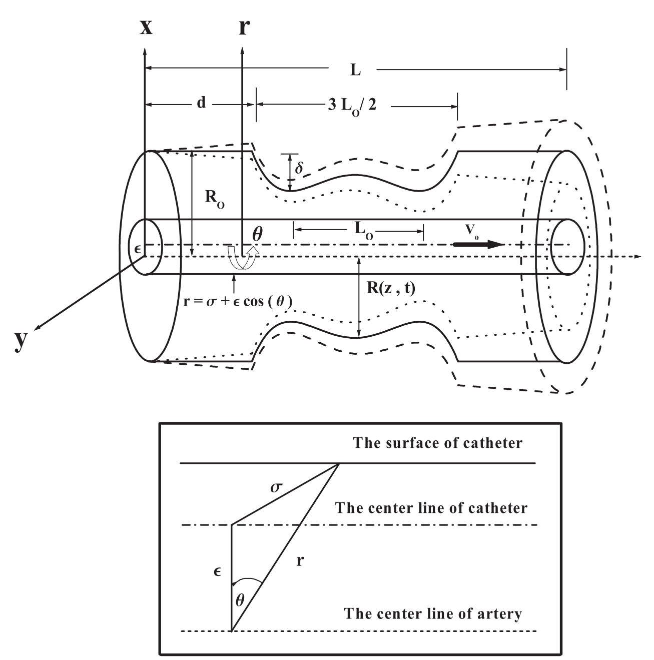

Fig. 2. Schematic diagram of eccentric catheterized overlapping stenosed artery.