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Analytical and numerical modeling of the transient elasto-dynamic response of a cylindrical tube to internal gaseous detonation

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Abstract

This paper reports the analytical and numerical modeling of the transient elasto-dynamic structural response of a cylindrical tube with finite length to internal detonation loading. The formulation of the proposed analytical model considers the effects of transverse shear and rotary inertia and also describes the effects of reflected waves. In the numerical part of this study, several transient-dynamic linear-elastic finite element analyses are carried out to obtain the structural response of the tube to pressure loads moving at different speeds. The results of the analytical and numerical simulations are compared with experimental results reported in the literature. It is shown that these models are capable of predicting the dynamic structural response of detonation tubes with a high degree of accuracy.

Keywords: Analytical model; Detonation tube; Elasto-dynamic response; Transient-dynamic finite element method

1. Introduction

There is significant interest in the development of reliable methods for design against damage caused by internal explosions in pressure vessels and pipes. This task requires a systematic understanding of the structural response of cylindrical tubes to internal detonation loading and often involves a combination of analytical, numerical, and experimental studies. On the other hand, the transient-dynamic nature of the process makes the problem very difficult.

Naturally, analytical models embody simplifications and assumptions, which often limit their ability to be used as reliable predictive tools [1–13]. For instance, the model of a thin-infinite shell proposed by Tang [3], starts with a general formulation, which considers the effects of rotary inertia and shear deformation but the outcome is confined to a *steady-state* solution. On the other hand, the model developed by Beltman and Shepherd [1,2] for a *finite* tube, considers *transient* behaviour, but ignores the rotary inertia and shear deformation effects. Recently, the development of a transient analytical model for a tube with finite length, which also includes the effects of rotary inertia and shear deformation, was reported by Mirzaei et al. [14]. The agreement between the predictions of the model and the experimental results of Beltman and Shepherd [1,2] was encouraging. However, the formulations did not include the effects of reflected waves. The current paper describes how the previous formulation can be extended to include these effects.

In the numerical part of this study, the finite element method (FEM) is used to obtain a more realistic modeling of geometry and boundary conditions. Several transient-dynamic linear-elastic FEM analyses are carried out to obtain the structural response of the tube to pressure loads moving at different speeds. Finally, the validity of the analytical and numerical simulations is investigated through comparisons with experimental results reported in the literature.

2. Gaseous detonation and structural response

In general, a gaseous detonation consists of a shock wave and a reaction zone that are coupled tightly. An ideal detonation travels at a nearly constant speed close to the Chapman–Jouguet velocity (V_{cj}), which is between 1500 and 3000 m/s in gases, depending on the fuel–oxidizer combination [1]. The pressure just behind the detonation can be as high as 20–30 times the ambient pressure. A schematic of a detonation tube along with a typical experimental pressure–time trace and an approximated profile for a detonation are shown in Fig. 1 [15].

The pressure history for this type of loading may be represented by an exponential approximation to the Taylor–

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Nomenclature

E	Young's modulus (N/m^2)	t_d	General time parameter (s)
F	loading function	V	load speed (m/s)
G	shear modulus (N/m^2)	V_{cj}	Chapman–Jouguet velocity (m/s)
H	step function	V_{d}	dilatational wave speed (m/s)
L	length of tube segments (m)	V_{s}	shear wave speed (m/s)
R	tube mean radius (m)	w	radial deflection (m)
T	exponential decay factor (s)	x	axial coordinate (m)
T_n	Time-dependent part of the solution	X_n	eigenmodes
h	tube thickness (m)	β	shell thickness parameter
p_1	pre-detonation pressure (Pa)	κ	shear correction factor
p_2	maximum-detonation pressure (Pa)	ν	Poisson's ratio
p_3	post-shock pressure (Pa)	ρ	density (kg/m^3)
p_{atm}	atmospheric pressure (Pa)	Λ_j	excitation parameter ($j=1, 2, 3$)
p_{cj}	Chapman–Jouguet pressure (Pa)	τ	time parameter (s)
n	mode index	ς	time parameter (s)
t	time (s)		

Zeldovich model and can be characterized by the initial pressure of the gas mixture p_1, the peak pressure p_2, the final pressure p_3, the exponential decay factor T, and the velocity V_{cj} as follows [1]:

$$p(t) = (p_1 - p_{\mathrm{atm}}) + [(p_3 - p_1) + (p_2 - p_3)e^{-t/T}] \times [1 - H(x - V_{\mathrm{cj}}t)]. \tag{1}$$

In the above equation, p_{atm} is the atmospheric pressure, x is the distance variable, t is the time variable, and H is the step function. From the structural point of view, the tube in which the detonation occurs experiences a traveling internal load that produces transient deformations. Fig. 2 shows the measured circumferential strain, as a function of time, for a tube subjected to detonation loading [1]. The strain history shows a sharp peak when the detonation passes the measurement point (at $t=$ 2.9 ms). For detonation loading, the circumferential strain can exceed the equivalent static strain (obtained for the tube under the static pressure p_{cj}) by a factor of up to 3–4. Such experimental results indicate that a simple static model of the tube cross-section is not sufficient for describing the dynamic nature of the strain distribution in the tube.

3. Analytical model

Before starting with the governing equations of the problem, it is beneficial to discuss specific features of the loading and boundary conditions. According to the schematic presented in

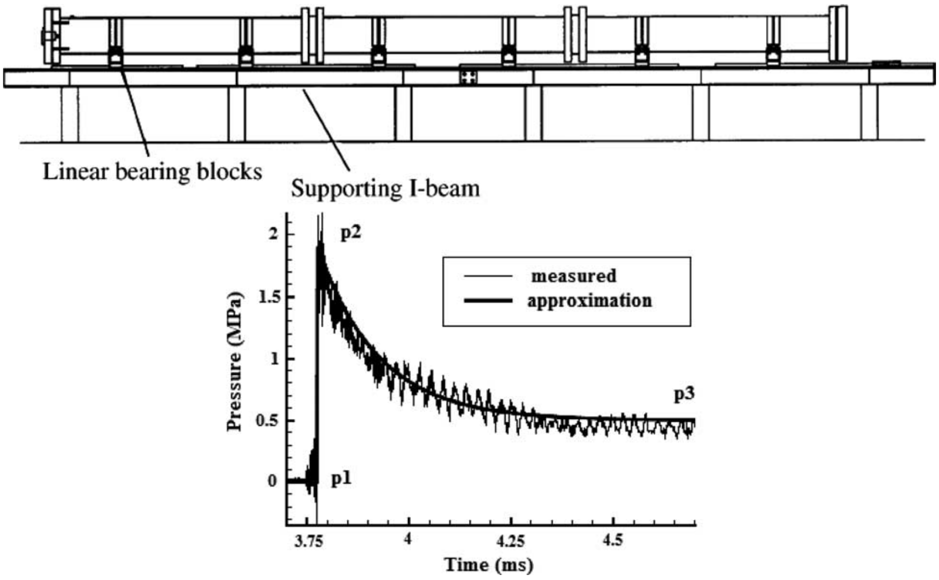

Fig. 1. (a) Schematic of a detonation tube [1]. (b) Comparison of measured and approximated pressure profiles. The oscillations in the measured pressure trace can be attributed to the structural vibrations of the pressure transducer mounts [15].